

# Stable running by leg force-modulated hip stiffness\*

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**Abstract**—Balancing the upper body as one of the main features in human locomotion is achieved by actuation of the compliant hip joints. Using leg force feedback to adjust the hip spring is presented as a new postural control technique. This method results in stable and robust running with the conceptual SLIP model which is extended by addition of a rigid trunk for upper body. Besides providing stability, this approach can represent the virtual pendulum (VP) concept which was observed in human/animal locomotion. Even more, the duality of this controller with virtual pendulum posture controller (VPPC) was mathematically shown. Such a mechanism could be also interpreted as a template for neuromuscular model.

## I. INTRODUCTION

Upright upper body is found in human [20] and animal locomotion [1]. Recently the virtual pendulum concept was proposed for postural control, based on experimental findings in human and animal locomotion [1]. It was shown that during stance phase, the ground reaction forces are intersecting in a virtual support point above center of mass (CoM), namely virtual pivot point (VPP) [1] or divergent point (DP) [2]. From control point of view, this concept can be employed to balance the trunk. Producing hip torque such that redirects the ground reaction forces to a predefined VPP could be an appropriate control approach. This technique was already utilized to generate stable walking/running [1]. Some extensions of the model to adjust the VPP in each step for robust hopping were developed [3], named virtual pendulum posture control (VPPC).

Stabilizing the gait and implementing the VP concept were already accomplished in walking [4], running and hopping [5] with a passive hip compliance. In the latter study, performance of hybrid zero dynamics (HZD) controller and VPPC were compared to a passive structure with compliant hip. It was shown that with combination of springs and damper in hip during stance phase, robust running and hopping with performance close to two other approaches can be achieved. Although mimicking the VPP with such a passive structure is an important step to implement VP concept mechanically, this approach has two drawbacks. First, the virtual pivot point produced by hip compliance could be placed close to the optimal location for stable and robust VPPC. However, there it is no established method to find the appropriate compliance characteristics to point out a specific

VPP in accordance to a pre-designed VPPC. Second problem comes from nonzero hip torques at take off and touch down which causes discontinuities in hip torque at these moments (switching between flight and stance phases). This second point prevents the proposed method to be applied as a controller in practice and deviates the hip torque from desired one, produced by VPPC or HZD. Although the conceptual models are not mechanically feasible, their controller can be extended to a physical model [6][7]. Having such large hip torques (the highest value in stance phase) at take off and touchdown does not match to the typically much smaller hip torque during swing phase[8].

In this study we want to resolve these drawbacks by using leg force feedback to adjust the stiffness. Variable stiffness were utilized for changing the natural dynamics of the system in order to attain different speeds or optimizing the energy consumptions like [9][10] and [11]. However, here the goal is representing a mechanism for tuning the hip stiffness in order to resembling a feature in human balance control. It is illustrated that an acceptable approximation of VPPC torque can be realized by hip compliance + leg force feedback. In that respect, the hip torque can mimic the activities of muscles between upper body and legs, like hamstring and rectus femoris.

## II. METHODS

### A. Simulation model

The simulation model which is used in this study is an extension of Spring Loaded Inverted Pendulum (SLIP) model with addition of a rigid trunk representing the upper body. In this model, called TSLIP [12] for Trunk-SLIP, the leg is modeled by a massless spring (like in SLIP) and the trunk represents a rigid upper body with mass  $m$  and moment of inertia  $J$  as shown in Fig. 1a. In [13] a similar model was introduced namely ASLIP, for “Asymmetric SLIP”. However, as this term can also designate a SLIP model with asymmetric leg properties, we prefer to use the appellation TSLIP. The model parameters are set to match the characteristics of a human with 80 kg weight and 1.89 m height (see Table I). Running dynamics (gait cycle) has two phases: *flight* and *stance*. Flight phase is described by the ballistic motion of the Center of Mass (CoM) when the leg does not touch the ground. The only control parameter in this phase is the leg orientation which can be arbitrarily adjusted, because the leg is massless. This angle has no effect on flight phase and just influences the touch down moment and configuration and

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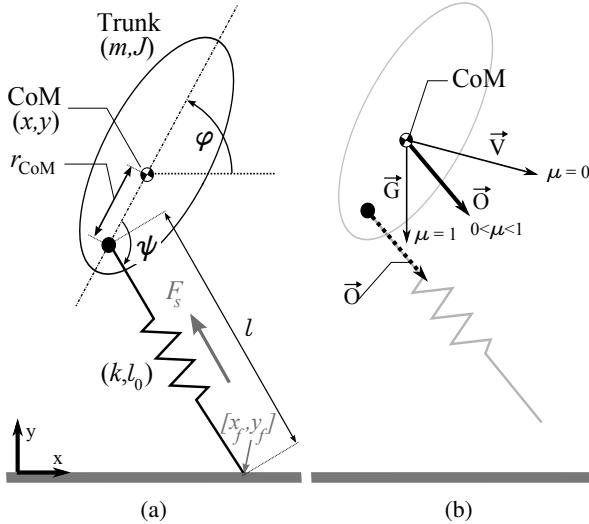


Fig. 1: (a) TSLIP model with a rigid trunk and a leg modeled as a massless prismatic spring. (b) Velocity-based leg adjustment (VBLA) during flight phase.

TABLE I: Model parameters

Parameter	symbol	value [units]
trunk mass	$m$	80 [kg]
trunk moment of inertia	$J$	4.58 [kg m <sup>2</sup> ]
distance hip-CoM	$r_{\text{CoM}}$	0.1 [m]
leg stiffness	$k$	16000 [N/m]
leg rest length	$l_0$	1 [m]
Nominal hopping/running height	$y^*$	2.5 [cm]

respectively, the motion in the next stance phase.

$$\begin{cases} m\ddot{x} = 0 \\ m\ddot{y} = -g \\ J\ddot{\varphi} = 0 \end{cases} \quad (1)$$

Stance phase starts by touchdown (TD), the moment that the distal end of the leg hits the ground and ends with takeoff (TO) when the  $\text{GRF} = [\text{GRF}_x \text{ GRF}_y]$  has no vertical component ( $\text{GRF}_y = 0$ ). In this phase,  $F_s = k(l_0 - l)$  gives the spring force along the leg axis, where  $l$ ,  $l_0$  and  $k$  are respectively the current leg length, leg rest length and the spring stiffness. Defining the states  $x$ ,  $y$  and  $\varphi$  as the CoM horizontal and vertical positions and the trunk orientation, respectively; the hip point ( $X_h = [x_h, y_h]$ ) which is positioned below CoM with distance  $r_h$  is obtained as follows

$$\begin{aligned} x_h &= x - r_h \cos \varphi \\ y_h &= y - r_h \sin \varphi \end{aligned} \quad (2)$$

The hip torque  $\tau$  is determined by the controller (VPPC or passively by compliant hip) for stabilizing the posture of the trunk during stance phase. The hip torque and the leg spring force produce the ground reaction force in interaction with the ground by

$$\begin{aligned} \text{GRF}_x &= F_s \frac{x_h}{l} + \frac{\tau y_h}{l^2} \\ \text{GRF}_y &= F_s \frac{y_h}{l} - \frac{\tau x_h}{l^2} \end{aligned} \quad (3)$$

Considering  $g$  as the gravity acceleration, the motion in the stance phase equations is described by

$$\begin{cases} m\ddot{x} = \text{GRF}_x \\ m\ddot{y} = \text{GRF}_y - g \\ J\ddot{\varphi} = \tau + r_h(\text{GRF}_x \sin \varphi - \text{GRF}_y \cos \varphi) \end{cases} \quad (4)$$

### B. Control approaches

For the TSLIP model, the controller is combined of leg adjustment in flight phase and hip torque control during stance phase as described in previous section. In leg adjustment, the leg angle during swing phase is controlled, while considering the massless leg simplifies it to determining the leg angle at touchdown moment, namely “angle of attack”. During the stance phase, a controller determines the required hip torque for stabilization of the motion, especially balancing the upper body. In the following, a short summary of the leg adjustment approach is presented and we concentrate more on trunk stabilization which is done by hip torque control.

1) *Leg adjustment during the flight phase:* The easiest leg adjustment approach is setting the leg angle to a fixed value. Although using a fixed angle of attack with respect to the ground can stabilize running [14] and walking [15], the region of attraction for the stable gait is quite small. This drawback which results in low robustness and sensitivity to running velocity changes and control parameters exist in other common leg adjustment methods (mostly based on Raibert approach [6]). In most of the leg adjustment strategies the foot landing position is adjusted based on the horizontal velocity [7] [16]. In this paper, VBLA (Velocity Based Leg Adjustment) presented in [12] is used as a robust method. This method can mimic human leg adjustment strategies for perturbed hopping [17] and achieve a large range of running velocities by a fixed controller [18]. In VBLA, the leg direction is given by vector  $\vec{\mathcal{O}}$  as a weighted average of the CoM velocity vector  $\vec{V}$  and the gravity vector  $\vec{G} = [0, -g]^T$  (Fig. 1b).

$$\vec{\mathcal{O}} = (1 - \mu)\vec{V} + \mu\vec{G} \quad (5)$$

with weighting constant  $\mu$  between 0 and 1.

2) *VPPC for hip torque control:* Intersection of ground reaction forces (GRF) during stance phase in a point above the CoM (VPP) was shown in human/animal walking and running [1]. This idea could be employed to design a controller producing hip torque which redirects GRFs toward a predefined VPP located above CoM (Fig. 2a). In that respect, the trunk behavior is transformed, from an inverted pendulum mounted at the hip to a regular virtual pendulum (VP) suspended at the VPP.

Knowing the leg spring force  $F_s$ , the hip torque  $\tau$ , required for producing the normal force ( $F_N$ ) to redirect GRF to VPP, is computed as follows (See Fig. 2a).

$$\tau = F_s l \frac{r_h \sin \psi + r_{\text{VPP}} \sin(\psi - \gamma)}{l + r_h \cos \psi + r_{\text{VPP}} \cos(\psi - \gamma)} \quad (6)$$

in which  $r_{\text{VPP}}$ ,  $\gamma$  and  $\psi$  are the distance between VPP and CoM, the VPP angle and the angle between the leg and the upper body, respectively. With this equation, balancing the

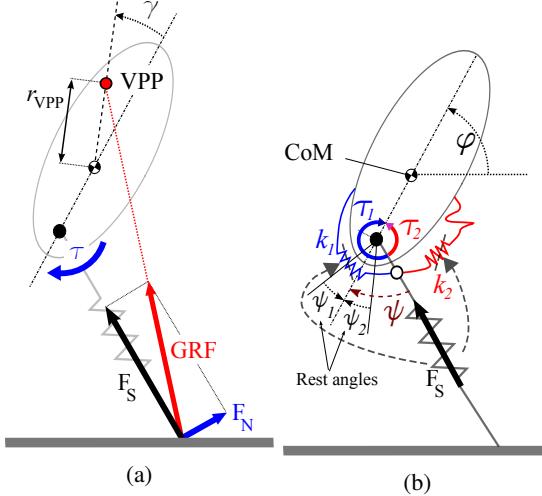


Fig. 2: (a) Virtual pendulum-based posture control (VPPC) during stance phase. (b) Hip spring adjustment, to implement VP.

upper body is performed, without measuring anything with respect to the environment, e.g. the absolute trunk orientation  $\varphi$ . The internal body sensors are sufficient to find the leg force  $F_s$  and angle  $\psi$  which are employed in the proposed controller (7). In the following, we set  $\gamma$  to zero to have the VPP on the trunk axis which is also sufficient for regular (without perturbation) human/animal locomotion [1]. This assumption simplifies (6) to

$$\tau = F_s l \frac{(r_h + r_{VPP}) \sin \psi}{l + (r_h + r_{VPP}) \cos \psi} \quad (7)$$

*3) Passive hip control:* In the passive compliant hip control approach, the hip torque  $\tau$  is produced by hip springs and damper. According to Fig. 2b, the two unidirectional springs work in opposite directions in different regions of angle between trunk and leg  $\psi$ . With constant values for rest angles  $\psi_1$  and  $\psi_2$ , stiffnesses  $k_1$  and  $k_2$  and damping ratio  $d$  the hip torque is computed by

$$\tau = k_1 \max(0, \psi - \psi_1) + k_2 \min(0, \psi - \psi_2) - d\dot{\psi}. \quad (8)$$

This mechanism represents human-like muscles, hamstring and rectus femoris<sup>1</sup>. The damping effect is considered to compensate the energy injected in each step by preloading the springs. It is removed when the leg force feedback is added in the following.

*4) Adapting hip compliance using leg force:* In the proposed passive compliant hip mechanism, one of the springs (with respect to the angle of attack) should be preloaded at touch down which means sudden increment in the energy of the system which makes the model physically infeasible. In order to mimic the hip torque patterns produced by VPPC, the leg force  $F_s$  is utilized as the feedback signal for hip

<sup>1</sup>These human muscles are biarticular which need two-segment leg. Hip springs can be interpreted as a mechanical representation of these muscles and can be extended in future to models with segmented legs.

spring stiffness ( $k_i$ ,  $i = 1, 2$ ) adjustment.

$$k_i = k_i^0 \frac{F_s^n}{F_s} \quad (9)$$

where  $k_i^0$ 's and  $F_s^n$  are the initial values of hip springs' stiffnesses and normalization value for the leg force, respectively. How this adaptive hip compliance can approximate VPPC<sup>2</sup> is described in Appendix. A. It was already suggested that the lengths of knee (vastus) muscle and biarticular muscles (hamstring and rectus femoris) correspond the virtual leg (the virtual line connecting hip to ankle) length and angle [19]. Therefore, this method presents a mechanical representation to implement the VP concept for postural stabilization of running, without requiring any extra measurements.

### C. Evaluating VPP existence in a controlled motion

As mentioned before, VPP is a concept which was observed in human/animal upper body balancing. For every control approach, existence of the VP concept can be investigated. VPP is defined [1] as “the single point at which the total transferred angular momentum remains constant and the sum-of-squares difference to the original angular momentum over time is minimal, if the GRF is applied at exactly this point”. In this paper, for the hip compliance (with or without leg force feedback) this point is found using the calculations described in Appendix. B. For every control approach, the existence of a VPP is given when the GRFs clearly intersect at a point above the center of mass.

## III. RESULTS

In this section, stability in running with VBLA for leg adjustment and VPPC, hip compliance with and without leg force feedback are investigated. As a standard model, TSLIP for running with parameters of Table I is simulated in MATLAB/SIMULINK 2012b using ode45 solver. The hip torque-angle behavior is analyzed and the ground reaction forces vectors during stance phase which demonstrate the VPP concept are also shown.

### A. Torque angle analysis

Stable running at 3m/s is achieved by a range of control parameters. Similar results are found for other speeds, just by changing the parameters and initial conditions. Hereafter, all hip control approaches are combined with VBLA for leg adjustment with  $\mu = 0.43$ . The first controller is VPPC with  $r_{VPP} = 8\text{cm}$  which can stabilize the motion. This value is selected based on finding a trade off between eigenvalue minimization and robustness maximization as explained in [3].

For the second approach, two different combinations are set for hip compliance which can stabilize the motion:

- Type 1 (overlap between springs working area): springs stiffnesses  $k_1 = k_2 = 300 \frac{\text{Nm}}{\text{rad}}$ , rest angles  $\psi_1 = -\psi_2 = -5^\circ$  and damping  $d = 0.5 \frac{\text{Nms}}{\text{rad}}$ .

<sup>2</sup>To resemble VPPC with VPP on the trunk axis, the rest angles may be set to zero.

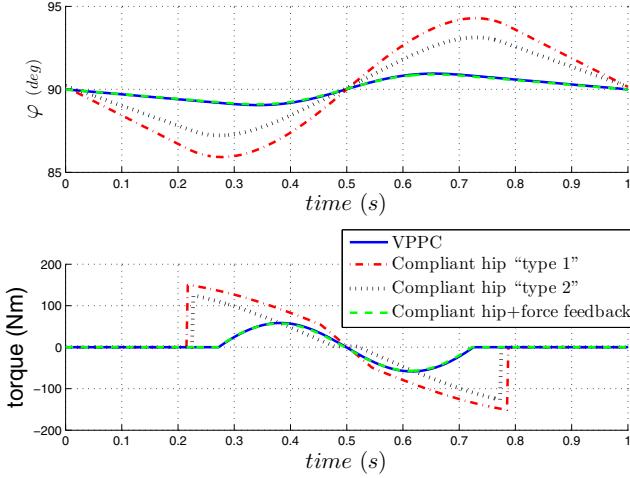


Fig. 3: Steady state (**top**) trunk angle and (**bottom**) hip torque trajectories during gait cycle.

- Type 2 (with dead zone): springs stiffnesses  $k_1 = k_2 = 350 \frac{Nm}{rad}$ , rest angles  $\psi_1 = -\psi_2 = 2^\circ$  and  $d = 0.5 \frac{Nm s}{rad}$ .

Leg force is utilized for hip spring stiffness adjustment using Eq. (9) with initial springs stiffnesses  $k_1^0 = k_2^0 = 250 \frac{Nm}{rad}$ , rest angles  $\psi_1 = -\psi_2 = 0^\circ$  without damping.

In Fig. 3, hip torque and trunk angle are shown during one step, from apex to apex. The gait duration is normalized to 1. It is observable that for all control types, the trunk angle with respect to ground  $\varphi$  does not deviate considerably from vertical orientation. In passive hip springs this deviation is less than  $4^\circ$  and for VPPC and hip spring + force feedback which are very close to each other, it is less than  $1^\circ$ . Such a low angular motion is also observed in human locomotion [20]. The trend is also similar to what humans do when after reaching apex (middle of flight phase), the upper body is slightly bending forward until touch down. Then, it starts leaning backward until takeoff and again moves forward until the next apex.

Fig. 3(bottom) illustrates the torque changes which has nonzero values during stance phase when no torque is needed for moving the massless leg in flight phase. For passive hip springs, jumps from zero to the highest produced hip torques, in addition to make the model impractical, make the trend different from VPPC and also human hip torque actuations, except around mid-stance. These drawbacks are resolved using leg force feedback which prepares torque pattern very similar to VPPC. This is expected from argumentation in Sec. II-B.4 and Appendix. A.

In Fig. 4, the hip torque is drawn versus the angle between upper body and leg ( $\psi$ ). This shows how the hip torque relates to hip angle. The dead zone and overlap of the springs are illustrated in this figure. It is conclude from this figure that with leg force feedback, it is possible to mimic VPPC torque-angle relation.

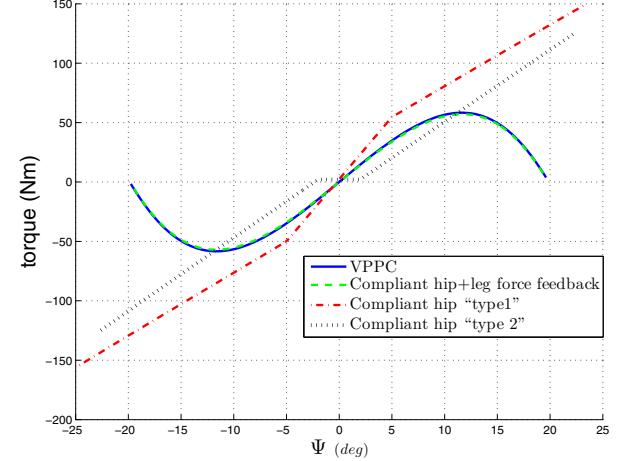


Fig. 4: Hip torque-angle diagrams for different control approaches.

#### B. VPP representation with Hip spring

The VPP is computed for the model with compliant hip (with and without leg force compliance) by the equations presented in Appendix. A. The computed VPP for different methods and the ground reaction forces are shown in CoM coordinate system in Fig. 5. Here, CoM is the origin and the ground reaction forces, originating at the center of pressure are displayed at different time instances. The estimated location of the VPP measured over running steps, is depicted by red point above the CoM. For the hip compliance type 1, the related  $r_{VPP}$  is  $34cm$  which is more than 4 times this value for VPPC. This distance is  $16.6cm$  for the second type of hip compliance which is about half of this value for the first type. Having dead zone in the middle of torque angle behavior makes both the motion behavior (Fig. 4) and VPP point closer to what achieved by VPPC. Although the GRFs almost intersect in VPP, it is not very precise. Adaptation of the hip spring stiffness via leg force both reduces the the related  $r_{VPP}$  and makes a more focused VPP from ground reaction forces. No out-layer forces like Figs. 5a and 5b exists in Fig. 5c.

#### IV. DISCUSSION

In this paper, an approach for implementing the virtual pendulum posture control (VPPC) is presented via addition of leg force feedback to tune the hip spring stiffness. In VPPC, trunk and leg angles are not required and just the angle between them should be known beside the leg force. On the other hand, we found useful properties of applying hip spring and damper in stabilizing hopping and running motion from our previous studies [5]. It was demonstrated that employing such a compliant hip gives similar torque patterns in the middle of stance phase and comparable robustness against perturbations with respect to VPPC. These two points of view concluded a need for tuning the hip spring to produce torque-angle behavior similar to VPPC in whole

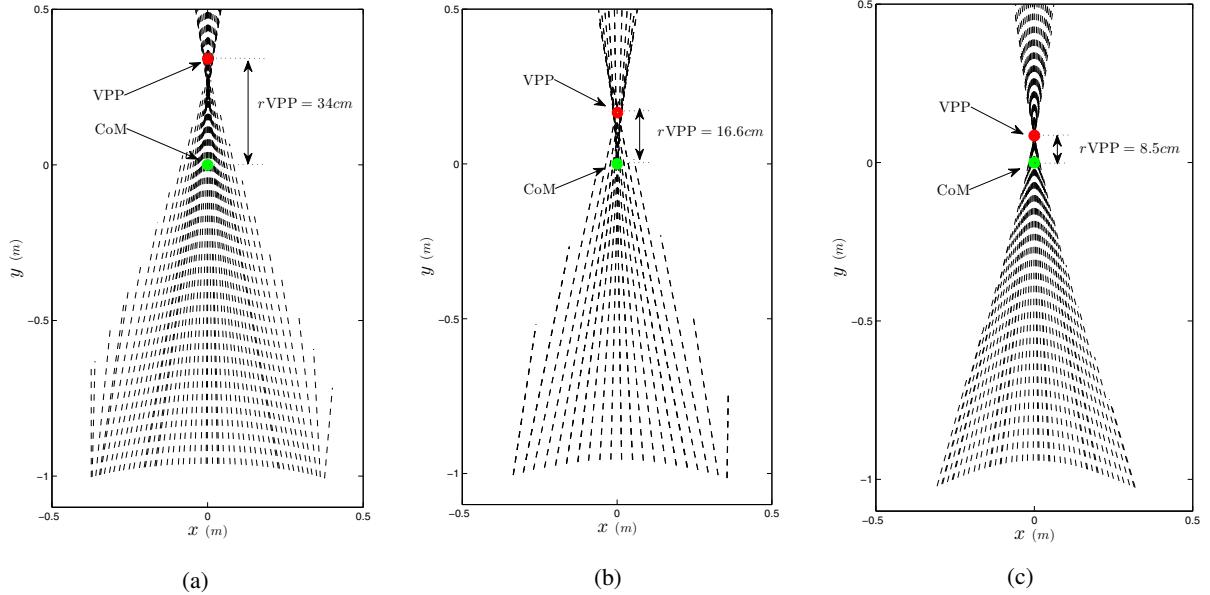


Fig. 5: VPP of running for a) compliant hip type 1 b) compliant hip type 2 c) hip spring+leg force feedback.

stance phase. In other words, we replaced damping and nonlinear spring relation with variable stiffness mechanism. In that respect, using the leg force feedback to adjust the hip spring stiffness helped to mimic the exact behavior of VPPC controller. Accordingly, with a passive mechanism beside an internal measurement of the leg property like leg length, a robust controller to balance the upper body is produced<sup>3</sup>. Existence of such kinds of sensors for measuring leg configuration in human body was already shown [19] (e.g. knee muscle measures the leg length).

By this method for hip control, in addition to reproducing the torque-angle pattern of VPPC, the exact VPP can be implemented with precise positioning of the intersection point of ground reaction forces. Therefore, to apply this controller in reality, the stabilizing and even robust VPPC controller can be designed and then, the parameters of the related mechanism for hip spring+leg force feedback can be obtained.

The idea of compliance adjustment was also introduced in Hill type muscle modeling [21] when it is triggered by an activation function  $A(t)$  as

$$F_m = A(t)F_l(l_m)F_v(\dot{l}_m)F_{max} \quad (10)$$

in which,  $l_m$  is the muscle length and  $F_m$ ,  $F_l$ ,  $F_v$  and  $F_{max}$  are the muscle force, the force-length relation, the force-velocity relation and the maximum isometric contraction force of the muscle, respectively. Therefore, the presented mechanism is like a Hill type muscle model for hip torque control which utilizes the leg force as activation function. Eq. (13) can be easily mapped to Eq. (10) considering spring relation with zero rest length for  $F_l$ , unity function for  $F_v$ ,  $F_{max} = \frac{1}{F_n^2}$  and  $A = F_s$ . The proposed functions for force-length and force-velocity relation were also suggested by Haeufle et al [22].

<sup>3</sup>The robustness is inherited from VPPC which was shown in [3].

The presented model could be interpreted as a muscle model with an activation function determined based on feedback signal of another muscle. For example, in human body, the hip muscles (hamstring and rectus femoris) may change their properties based on vastus muscle length which measures the leg length. The function of such a biologically motivated hip control could be simulated based on a series-elastic-actuator concept as proposed by [23].

In this paper the focus was on establishing new, biologically plausible control mechanisms for upright trunk posture in locomotion. Based on an extension of the conceptual SLIP model, we were able to identify that leg force feedback is appropriate to tune compliant hip function. This finding can be translated into neural circuits between different leg muscles helping to establish balance during locomotion. Similar neuro-muscular networks have been previously suggested for repulsive leg function (e.g. in bouncy tasks like hopping, [24]). The similarity of the proposed networks for balance and spring-like leg function suggest that different functional requirements for locomotion could be implemented by applying similar sensor-motor relations to different connections (e.g. between single-joint and two-joint muscles for balance) within the neuro-muscular system. In this respect, the VPP approach proved to be a very useful “navigation tool” to identify appropriate control schemes, which could be used for both technical and biological systems.

## APPENDIX

### A. Approximation of VPPC with hip compliance+leg force feedback

Considering  $r$  as the distance between VPP and hip which means ( $r = r_{VPP} + r_h$ ), from Eq. (7) the hip torque produced

by VPP is obtained by:

$$\tau_{\text{VPP}} = F_s l \frac{r \sin \psi}{l + r \cos \psi} \quad (11)$$

For angles  $\psi$  less than  $30^\circ$  the error of approximating  $\sin \psi$  with  $\psi$  is less than 6%. In human, the distance between the CoM of upper body and the hip ( $r_h$ ) is less than 10% of the leg length [25]. The VPP distance ( $r_{\text{VPP}}$ ) is also not more than 20 cm which is around 20% of the leg length [1]. In our simulation the optimal value was 8 cm. When difference between  $\cos \psi$  and one is less than 0.125 for  $\psi$  smaller than  $30^\circ$ , approximating  $l + r \cos \psi$  with  $l + r$  does not make error more than 3%. It is remarkable that these approximation errors happen in the same direction (both are positive or negative) which reduce total error of the following approximation to less than 1.5%. With such a close approximation, the VPP torque can be written as:

$$\tau_{\text{VPP}} \approx F_s l \frac{r \psi}{l + r} \quad (12)$$

On the other hand, setting  $\psi_1 = \psi_2 = 0$  and combining Eqs. (8) and (9) gives the following hip torque relation for hip spring+leg force feedback.

$$\tau_h = \begin{cases} k_1^0 \frac{F_s}{F_s^n} \psi & \psi > 0 \\ k_2^0 \frac{F_s}{F_s^n} \psi & \psi < 0 \end{cases} \quad (13)$$

It can be easily seen that using the following equation for initial stiffness  $k_i^0$ , equalizes the hip torques in two methods ( $\tau_h$  and  $\tau_{\text{VPP}}$ ).

$$k_i^0 = \frac{lr}{(l+r)} F_s^n \quad (14)$$

Note that, Eq. (9) demonstrates a kind of interpretation of normalizing the leg force and considering zero rest angles, the leg force modulated compliance torque may also be written as follows.

$$\tau_h = \frac{lr}{(l+r)} F_s \psi = k(F_s) \psi \quad (15)$$

Which means the hip actuator is a compliance having variable stiffness with a linear relation to leg force.

### B. Finding VPP during stance phase

First of all, we need to compute the GRFs in the coordinate system centered at CoM and with vertical axis in trunk orientation. Then defining  $[x, y]$ ,  $[x_f, y_f]$  and  $\varphi$  as the position of CoM, foot contact and the trunk angle (See Fig. 1a), the foot point in the new coordinate system ( $P^c := [x_f^c, y_f^c]$ ) is computed by

$$\begin{cases} x_f^c &= (x_f - x) \sin \varphi - (y_f - y) \cos \varphi \\ y_f^c &= (x_f - x) \cos \varphi - (y_f - y) \sin \varphi \end{cases} \quad (16)$$

Also the GRF in the new coordinate system ( $F^c := [F_x^c, F_y^c]$ ) is computed as follows:

$$\begin{cases} F_x^c &= \text{GRF}_x \sin \varphi - \text{GRF}_y \cos \varphi \\ F_y^c &= \text{GRF}_x \cos \varphi - \text{GRF}_y \sin \varphi \end{cases} \quad (17)$$

The torque generated by  $F^c$  exerted at  $P^c$  around the origin (CoM) is computed by outer product ( $\times$ ) of these two vectors:

$$\tau^c = F_x^c y_f^c - F_y^c x_f^c = F^c \times P^c \quad (18)$$

Assume that vectors  $\vec{F}_x$ ,  $\vec{F}_y$ ,  $\vec{X}$  and  $\vec{Y}$  are composed of concatenating the related values  $F_x^c$ ,  $F_y^c$ ,  $x_f^c$  and  $y_f^c$  during stance phase in 4 columns, respectively. Then, the summation of torques produced by GRFs during stance phase is obtained by

$$\tau = \vec{F}_y^T \vec{Y} - \vec{F}_x^T \vec{X} \quad (19)$$

where upper index  $(.)^T$  stands for transpose. The first condition in existence of VPP is having a constant total transferred angular momentum [1] which should be equal to  $\tau$ . Suppose that vector  $\vec{F} = [F_x, F_y]$  represents the average of GRFs during stance phase. Each point on a line  $l$  with slope  $\frac{F_y}{F_x}$  and distance from origin  $d = \frac{\tau}{|\vec{F}|}$  has total transferred angular momentum equal to  $\tau$ . This line is defined by  $l : y = ax + b$  where  $a$  and  $b$  are as follows:

$$\begin{cases} a = \frac{F_y}{F_x} \\ b = \text{sign}(F_x)d\sqrt{1+a^2} \end{cases} \quad (20)$$

Exerting force  $\vec{F}$  from any point on this line produces torque  $\tau$ . For the second condition, the VPP point should be found such that applying the GRF at that point minimizes the sum-of-squares difference to the original angular momentum over time. With some mathematical manipulation, the VPP is obtained by the following relation:

$$\begin{cases} x_{\text{VPP}} = (\alpha^T \alpha)^{-1} \alpha^T \beta \\ y_{\text{VPP}} = ax_{\text{VPP}} + b \end{cases} \quad (21)$$

in which considering “.” for element-wise multiplication of two vectors,  $\alpha$  and  $\beta$  are defined as follows:

$$\begin{cases} \alpha := \vec{F}_x a - \vec{F}_y \\ \beta := \vec{F}_x \cdot (\vec{Y} - b) - \vec{F}_y \cdot \vec{X} \end{cases} \quad (22)$$

## REFERENCES

- [1] H. M. Maus, S. Lipfert, M. Gross, J. Rummel, and A. Seyfarth, “Upright human gait did not provide a major mechanical challenge for our ancestors,” *Nature Communications*, vol. 1, no. 6, pp. 1–6, 2010.
- [2] K. G. Gruben and W. L. Boehm, “Force direction pattern stabilizes sagittal plane mechanics of human walking,” *Human Movement Science*, vol. 31, no. 3, 2012.
- [3] M. A. Sharbafi, C. Maufroy, M. N. Ahmadabadi, M. J. Yazdanpanah, and A. Seyfarth, “Robust hopping based on virtual pendulum posture control,” *Bioinspiration and Biomimetics*, vol. 8, no. 3, 2013.
- [4] J. Rummel and A. Seyfarth, “Passive stabilization of the trunk in walking,” in *International Conference on Simulation, Modeling and Programming for Autonomous Robots*, Darmstadt, Germany, 2010.
- [5] M. A. Sharbafi, M. N. Ahmadabadi, M. J. Y. A. Mohammadinejad, and A. Seyfarth, “Compliant hip function simplifies control for hopping and running,” in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2013)*, 2013.
- [6] M. H. Raibert, *Legged Robots that Balance*. MIT Press, Cambridge MA, 1986.
- [7] I. Pouliquen and J. W. Grizzle, “The spring loaded inverted pendulum as the hybrid zero dynamics of an asymmetric hopper,” *IEEE Transaction on Automatic Control*, vol. 54, no. 8, pp. 1779–1793, 2009.

- [8] S. J. Piazza and S. L. Delp, "The influence of muscles on knee flexion during the swing phase of gait," *Journal of Biomechanics*, vol. 29, no. 6, pp. 723–33, 1996.
- [9] B. Vanderborght, B. Verrelst, R. V. Ham, M. V. Damme, D. Lefever, B. M. Y. Duran, and P. Beyl, "Exploiting natural dynamics to reduce energy consumption by controlling the compliance of soft actuators," *The International Journal of Robotics Research (IJRR)*, vol. 25, no. 4, pp. 343–358, 2006.
- [10] I. Thorson, M. Svinin, S. Hosoe, F. Asano, and K. Taji, "Design considerations for a variable stiffness actuator in a robot that walks and runs," in *In Proc. of the Robotics and Mechatronics Conference (RoboMec)*, 2007.
- [11] J. W. Hurst, "The role and implementation of compliance in legged locomotion," Ph.D. dissertation, Robotics Institute, Carnegie Mellon University, 2008.
- [12] M. A. Sharbafi, C. Maufroy, A. Seyfarth, M. J. Yazdanpanah, and M. N. Ahmadabadi, "Controllers for robust hopping with upright trunk based on the virtual pendulum concept," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (Iros 2012)*, 2012.
- [13] I. Poulakakis and J. Grizzle, "Formal embedding of the spring loaded inverted pendulum in an asymmetric hopper," in *in Proc. of the European Control Conference*, Kos, Greece, 2007.
- [14] A. Seyfarth, H. Geyer, M. Guenther, and R. Blickhan, "A movement criterion for running," *Journal of Biomechanics*, vol. 35, no. 5, pp. 649–655, 2002.
- [15] H. Geyer, A. Seyfarth, and R. Blickhan, "Compliant leg behaviour explains basic dynamics of walking and running," *Proceedings of the Royal Society B*, vol. 273, no. 1603, pp. 2861–2867, 2006.
- [16] A. Sato and M. Beuhler, "A planar hopping robot with one actuator: Design, simulation, and experimental results," in *4 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2004.
- [17] M. A. Sharbafi and A. Seyfarth, "Human leg adjustment in perturbed hopping," in *AMAM*, 2013.
- [18] M. A. Sharbafi, M. N. Ahmadabadi, M. J. Yazdanpanah, and A. Seyfarth, "Novel leg adjustment approach for hopping and running," in *Dynamic Walking*, 2013.
- [19] R. Desai and H. Geyer., "Muscle-reflex control of robust swing leg placement," in *IEEE International Conference on Robotics and Automation, (ICRA 2013)*, 2013.
- [20] E. Muybridge, *The Human Figure in Motion*. New York: Dover Publications Inc, 1955.
- [21] A. V. Hill, "The heat of shortening and the dynamic constants of muscle," *Proceedings of the Royal Society B*, vol. 126, 1938.
- [22] D. F. B. Haeufle, S. Grimmer, and A. Seyfarth, "The role of intrinsic muscle properties for stable hopping stability is achieved by the force-velocity relation," *Bioinspiration and Biomimetics*, vol. 5, no. 1, 2010.
- [23] D. W. Robinson, J. E. Pratt, D. J. Paluska, and G. A. Pratt, "Series elastic actuator development for a biomimetic walking robot," in *IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, 1999.
- [24] H. Geyer, A. Seyfarth, and R. Blickhan, "Positive force feedback in bouncing gaits?" *Proceedings of the Royal Society B*, vol. 270, 2003.
- [25] D. A. Winter, *BioMechanics and motor control of human movement*, 3rd ed. New Jersey, USA: John Wiley & Sons, Inc, 2005.