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TIP Model: A Combination of Unstable Subsystems for Lateral Balance in Walking

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Abstract— **Balancing or postural stability is one of important locomotor subfunctions in bipedal gaits. The inverted pendulum and virtual pivot point (VPP) are common modeling approaches to analyze balance control in human and robot walking. In this paper, we employ the VPP concept to investigate posture control in the frontal plane. The outcomes demonstrate that unlike posture control in the sagittal plane, the VPP in the frontal plane is place below center of mass. This finding explains a novel hybrid strategy for lateral stability in human walking. The here proposed model shows that switching between unstable inverted virtual pendulums generate stable posture control in the frontal plane. This outcome is consistent within a group of seven human subjects walking at normal and slow speeds.**

I. INTRODUCTION

Bipedal locomotion is defined by a hybrid dynamical system comprised of not only different dynamics (phases), but also combination of intensely coupled sub-systems for each phase. Inspired by biological legged locomotion and conceptual modeling (e.g., using template models [1]), this complex problem can be divided into three locomotor subfunctions (Fig. 1) [2]: *Stance:* the axial function of the stance leg that directs the center of mass (CoM) by applying force on the ground. *Swing:* Movement of the leg from toe-off to touch-down. *Balance:* Body posture control. This categorization was also employed to control robots [3], [4]. At first glance, these subsystems can be controlled separately. However, interaction between subfunctions is critical to achieve an efficient gait with acceptable robustness against perturbations [5].

As most of the human weight is in the upper extremity and it's supporting base is small, human walking is inherently unstable [6], [7]. Therefore, unlike quadrupedal locomotion, balance control is critical for bipedal gait e.g., human walking [7]. Nevertheless, using template models [1] such as spring loaded inverted pendulum (SLIP) model [8], locomotion can be represented with simplified mechanical oscillators [9]. In addition, it is shown that by using simple mechanical designs, a stable gait can be created [10]. Passive dynamic walkers can walk on a shallow slope without energy consumption. However, these models are limited to the sagittal plane. Because of incompatibility between the roll velocity and ground contact condition, the passive dynamic walker in the frontal plane is unstable [11].

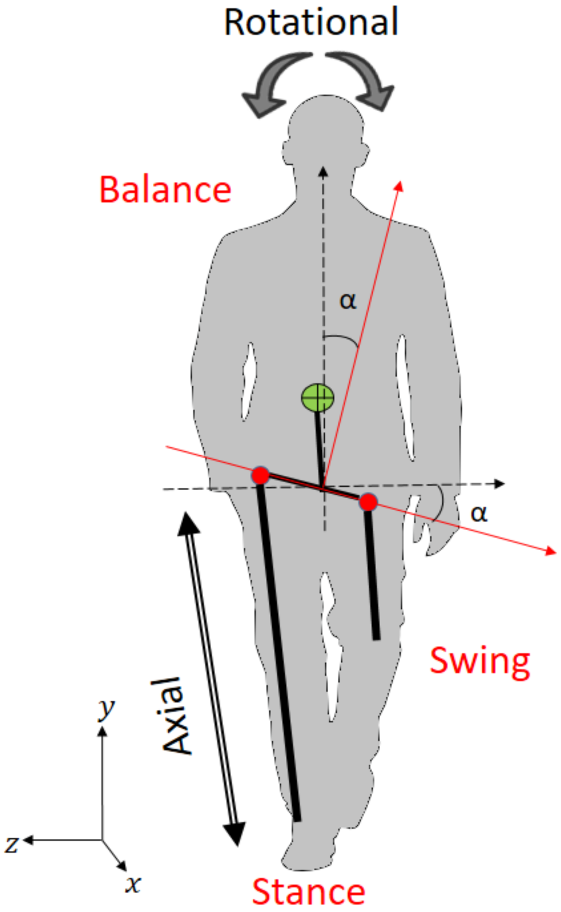

Fig. 1: Main locomotion sub-functions in frontal plane i) Stance: axial leg function ii) Swing: Rotational free leg swinging iii) Balance: posture control. Pelvis orientation (defined by α) is considered for body posture alignment.

In [11], several methods have been presented for lateral stabilization. Among these methods, lateral foot placement uses lower energy than others and wider steps result in lower instability. That's why elderly people prefer to take wider steps compared to youngsters [11]. This can be one strategy to compensate lateral instability, whereas taking wider steps will increase energy consumption [12][13]. In [14], balance control in the frontal plane is achieved using curved foot. In all of these methods, change in direction of the ground

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reaction force (GRF) in the frontal plane results in stability.

Skipping balance control by reducing the upper body to a concentrated point mass, simplifies stability to swing leg adjustment while the stance leg is modeled by an inverted pendulum [11]. In a more recent study, Maus et al. introduced a virtual pendulum (VP) model for posture control while including an extended upper body [15]. The VP concept states GRF vectors intersect at a point in the body reference frame called virtual pivot point (VPP). In the sagittal plane this point is above the CoM. Placing the VPP above the CoM can act like a regular pendulum which defines the so called virtual pendulum which is neutrally stable. By addition of a rigid trunk to the SLIP model (in the TSLIP model [16]) and using the VPP concept for posture control stable walking addressing all three locomotor subfunctions can be predicted [15]. This TSLIP model was used in different studies to investigate the effect of the upper body on motion stability and robustness against perturbations [15], [17], [18], [19]. For posture control the upper body orientation with respect to ground can be used [20][21] while in the VPP-based methods, the relative angle between the upper body and the leg is employed for control [15]. This is crucial to implement the control concept using mechanical compliant elements such as adjustable springs (e.g., in force modulated compliant hip (FMCH) model [17]) which can mimic human muscle behavior.

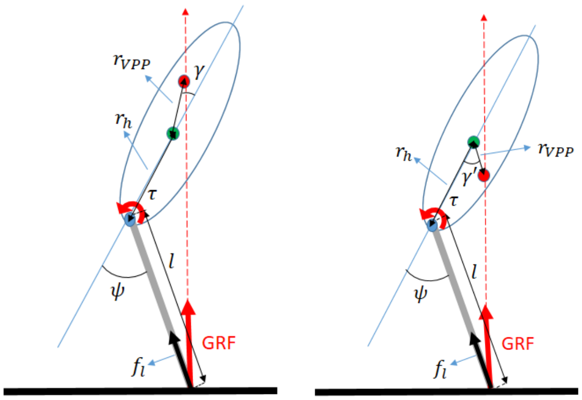
In this study, we will use the VPP concept in the form of three inverted pendulum (TIP) to describe human lateral stability for the first time. Although the control technique is completely different from the one used in sagittal plane, we show that the VPP concept is a useful tool to understand balancing in frontal plane. The rest of this paper is organized as follows, Sec. II explains the methods including the required information about VPP concept, its calculation for predicting human walking behavior in frontal plane, and description of data analyses. Sec. III describe the experimental and simulation results. Discussion about lateral stability using the template models is presented in Sec. IV.

II. METHODS

The VP (Virtual Pendulum) concept is an observation emerged from human balance control. From this perspective, this is not originally a control method, but the concept can be used to design posture control in humanoid robots or assistive devices. In this section, we describe how to verify the VPP in a stable bipedal gait and how it can be used to control within a simple model.

A. Calculation and interpretation of VPP

As stated in [15], VPP is defined as: *A single point that i) the total angular momentum remain constant and ii) the sum-of-square difference to the original angular momentum over time is minimal, if the GRF applied at exactly this point.* To find the location of VPP, we need a reference frame which is specified with respect to human body. A reference frame is specified by the location of its origin and its angular orientation. A reference frame with respect to the body should be carefully specified. Fixed reference frames such as the world coordinate frame cannot be used to find the location of VPP because the body moves at different speeds while walking. Also, Body shape changes along walking, that is why there are various reference frames that might have a mechanical or neural significance [22]. In [15] CoM has been used as an origin, and frame orientation has been aligned with the torso orientation. Gruben has shown that a reference frame attached to the CoM and aligned with the whole body direction in the sagittal plane could estimate whole-body angular momentum in a an acceptable manner [22].



(a) (b)

Fig. 2: VPP concept for posture control. Red and green points indicate the VPP and the CoM, respectively. a) VPP above the CoM, b) VPP below the CoM.

In order to identify existence of VPP in a stable gait, we need to draw the ground reaction forces (GRF) in the coordinate frame which represents upper body orientation. If a focused intersection point of GRFs is clearly observable, one can say the VPP concept is valid for analyzing posture control of that specific gait [15]. The position of VPP with respect to the CoM determines the template model (inverted or regular pendulum) to analyze posture control. When VPP is placed above the center of mass (Fig. 2a), the posture control can be predicted with a regular pendulum as a dynamic system with neutrally stable periodic orbits. If the GRF vectors intersecting below the CoM, balancing can be modeled by an inverted pendulum that is inherently unstable (Fig. 2b).

By now, the VP concept has been used for postural control in the sagittal plane in which the VPP is above CoM for normal walking. Using the TSLIP model (Trunk+SLIP [18]) shown in Fig.2a, the required torque (τ_{VPP}) for crossing the GRF vectors at a virtual pivot point which is located above the CoM is given by:

$$\tau_{VPP} = F_l l \frac{r_h sin\psi + r_{VPP} sin(\psi - \gamma)}{l + r_h cos\psi + r_{VPP} cos(\psi - \gamma)} \qquad (1)$$

in which F_s, ψ, r_{VPP} and γ are the leg force, the angle between upper-body orientation and the virtual leg, the VPP distance to CoM and deviation angle from trunk axis, respectively, as shown in Fig. 2a. If the VPP is below CoM, then $90^\circ < |\gamma|$. In this condition, replacing γ with γ' as shown in Fig. 2b turns Eq. (2) into

$$\tau_{VPP} = F_l l \frac{r_h sin\psi - r_{VPP} sin(\psi - \gamma')}{l + r_h cos\psi - r_{VPP} cos(\psi - \gamma')} \tag{2}$$

In both test cases shown in Fig. 2, the generated hip torque will reduce trunk orientation. This means that both pendulum or inverted pendulum models of describing the relation between CoM and VPP, predict stabilizing upper-body posture. Furthermore, other conditions can be demonstrated that both regular and inverted pendulum models will result in destabilizing the upper body. The relation between virtual (inverted) pendulum model and lateral balancing will be described more in Sec. IV.

B. *VPP for lateral stability*

Due to the narrow width of the base of support during the single support phase, keeping lateral stability is more difficult than keeping stability in the sagittal plane [23]. Also, due to the critical role of the pelvis in the frontal plane, lateral stability is different from stability in sagittal plane.

To find VPP in the frontal plane, we use two reference frames. As net external force, which is applied to the body, causes acceleration in center of mass, this point is a logical choice for the origin [22]. Therefore, we select the whole body CoM (WBCoM) and the upper body CoM (UBCoM) as the origins for the two coordinate systems. Balancing the torso around the supporting leg is dependent on the control of pelvic motion by the hip musculature and the coupling between the pelvis and the trunk. Since the weight of the trunk acts downward through the pelvis, the pelvic motion is critical to the maintenance of total body balance [23]. Hence, the pelvis direction can determine the reference frame direction. Then, in both cases, y-axis is aligned with the pelvis orientation as defined in OpenSim (see Sec. II-C). The VPPs are calculated for the right single support (RSS), double support (DS) and the left single support (LSS) separately.

C. *Data analyses*

In this study, we have used motion capture data from 7 male individuals (age: 25 ± 5 years, height: 1.86 ± 0.04 m, mass: 84 ± 15 kg; mean ± standard deviation). Tab. I presents the subject-specific experimental data. This data that is available on [24] has the following conditions:

1) The speed is selected freely by the human subject. (1.45 ± 0.15 m/s).

2) Approximately 80% of the free speed is called slow speed (1.2 ± 0.1 m/s).

For each of the aforementioned conditions, the subjects completed at least 3 overground trials in which the optical marker trajectories, ground reaction forces and moments were measured. Using 8-camera optical motion capture system, the trajectories of 41 markers were collected at 100 Hz (Motion Analysis Corp., Santa Rosa, CA, USA). In addition, the ground reaction forces and moments were collected at 2000 Hz from 3 floor-mounted force plates (Bertec Corp., Columbus, OH, USA). To calculate inverse kinematics, we use OpenSim software in accordance with the following work flow. OpenSim is an open source software to simulate the musculoskeletal system [25][26].

We employ a fully three-dimensional musculoskeletal body model which contains 39 degrees of freedom [27]. First, using the OpenSim Scale Tool, we scaled the geometry of the generic musculoskeletal model to match the anthropometry of each subject. Then, we use the Inverse Kinematic Tool to calculate the joint angle trajectory for each trial. The OpenSim's Residual Reduction Algorithm (RRA) Tool is utilized to refine the model kinematics so that they are more dynamically consistent with the experimental reaction forces and moments [25]. Finally, we use the RRA output kinematics to calculate the position of the CoM and pelvis angle.

TABLE I: Subject-specific experimental data from [24].

Subject	mass [kg]	height [m]	age	number of trials	
				slow speed	free speed
1	112.4317	1.9177	27	3	6
2	89.2338	1.8796	20	5	5
3	86.9085	1.905	19	7	7
4	64.0353	1.8034	21	6	5
5	85.0668	1.828	31	7	5
6	67.1419	1.8288	32	7	7
7	83.7903	1.828	27	5	5

III. RESULTS

In this section, we present our analyses on lateral balancing based on VPP concept. This includes investigating VPP in frontal plane at different phases (single and double support) as well as the changes in VPP position with respect to the motion speed. To calculate VPP, we use WBCoM or UBCoM as the origin of the coordinate system while in all cased the y-axis is always set by the trunk orientation (the line perpendicular to the pelvis).

A. *Identification of VPP in frontal plane*

For investigating VPP in different conditions, the ground reaction forces (GRF) should be drawn in the body coordinate frame. In Fig. 3, the GRFs of one walking step at preferred walking speed are plotted from CoP by dashed lines in the WBCoM-centered coordinate system. This figure illustrates an example of how clear the VPP is formed with respect to this reference frame. In all three phases (right single support (RSS), double support (DS) and left single support (LSS), the VPP is placed below CoM. This shows that switching three unstable subsystems (inverted pendulums) generates stable posture control in frontal plane. Similar behavior is observed in the UBCoM-centered coordinate frame as shown in Fig. 4. These two figures show how upper body and whole body posture is controlled in the frontal plane.

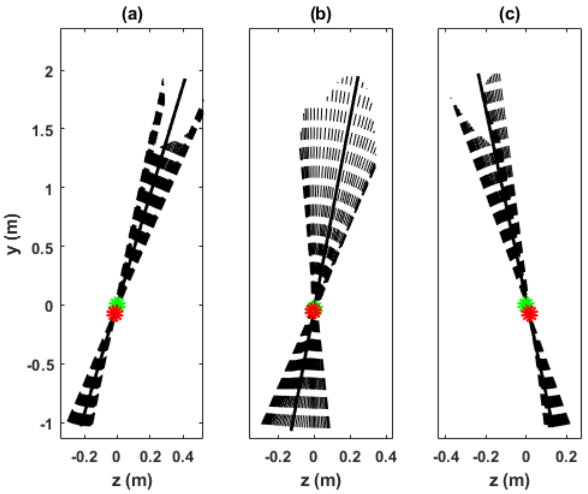


Fig. 3: The VPP (green circle) in WBCoM-centered (red circle) coordinate frame aligned with trunk orientation. Dashed lines show the ground reaction forces from CoP. a) Left single support b) Double support c) Right single support

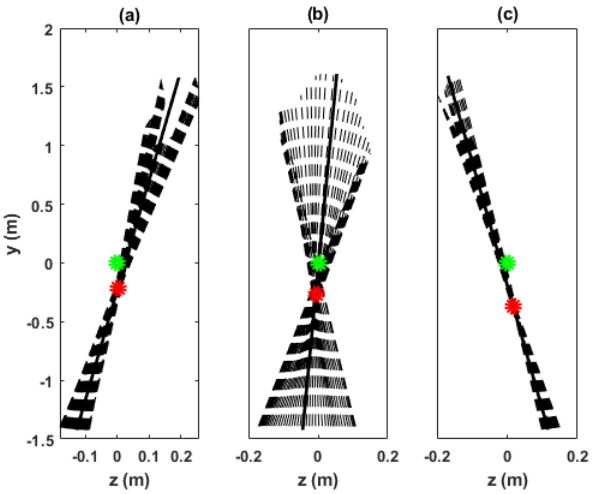


Fig. 4: The VPP (green circle) in UBCoM-centered (red circle) coordinate frame aligned with trunk orientation. Dashed lines show the ground reaction forces from CoP. a) Left single support b) Double support c) Right single support

B. *Variation of VPP in different conditions*

In this section, we will investigate the position of VPP in different walking phases at different speeds. The results are presented at preferred speed and slow speed for two aforementioned reference frames. Fig. 5 shows the mean and variance of the VPP's vertical position for walking at a freely selected speed and slow speed in WBCoM-centered frame. It is noticeable that vertical position of VPPs in single support is always below the body CoM (WBCoM). This is also true for double support phase except for one subject at free speed and 3 subjects at slow speed. These results show that whole body lateral balance control is achieved by switching between unstable inverted pendulum-like movements. It is also important that the variation of VPP position at different steps is small which shows consistent behavior in keeping VPP for lateral balance. Comparison between VPP in single support and double support shows that except for subject 2, the VPP of single support is lower than that of double support. Analyzing kinematic patterns of this subject (not shown here) depict inclination in the body posture during whole gait cycle.

Similar results are shown in UBCoM-centered coordinate frame in Fig. 6. Regarding the vertical position of VPP with respect to the upper body CoM (UBCoM), the results are more consistent as all values for single and double support have negative values. Therefore, upper body always behave as an inverted pendulum connecting to the VPP and switching between these unstable behaviors produces a stable lateral balance as explained in previous section (Fig. 3 and Fig. 4). Comparing the outcomes of WBCoM-centered frame (Fig. 6) with those of UBCoM-centered frame (Fig. 6) shows that the variance of the VPP vertical position of the whole body is less than those of the upper body at both walking speeds.

Another observation is the relation between the VPP height and walking speed in each case. As shown in Fig. 5 and 6, in the double support phase, the height of VPP position are lowered for all subjects as walking speed increases (except subject 1). This subject has a different behavior which might relate to his body properties as his Body Mass Index (BMI) is totally different from others. In the single support phase the relation between VPP position and motion speed is not as clear as in the double support phase. This might relate to unequal roles of different legs depending on the dominant foot which can also explain the difference in VPP heights in the left and right legs.

Due to the distance between the left and right hip in the frontal plane, investigating the horizontal position of VPP with respect to the center of mass can provide useful information. In Fig. 7, these values are depicted at free speed, in the two aforementioned coordinate frames. In the WBCoM-centered frame, in the left single support phase, VPP is on the left side of WBCoM, and in the right single support phase, VPP is on the right side of WBCoM. This pattern is valid UBCoM-centered coordinate frame except for right single support of subject 1 and 2.

In all experiment trials except for subject 4, the stride starts with right single supports (RSS). Fig. 7 shows that the horizontal position of VPP with respect to CoM in double support (DS) phase relates to transition order of the legs (from right to left or vice versa) except for subject 1. This was the subject who had positive VPP height in the WBCoM-centered frame. Therefore, if the double support phase starts with touchdown of the left leg (order: RSS->DS->LSS), the VPP of the DS will be in the left side of the CoM (similar to that of LSS). More details will be described in Sec. IV.

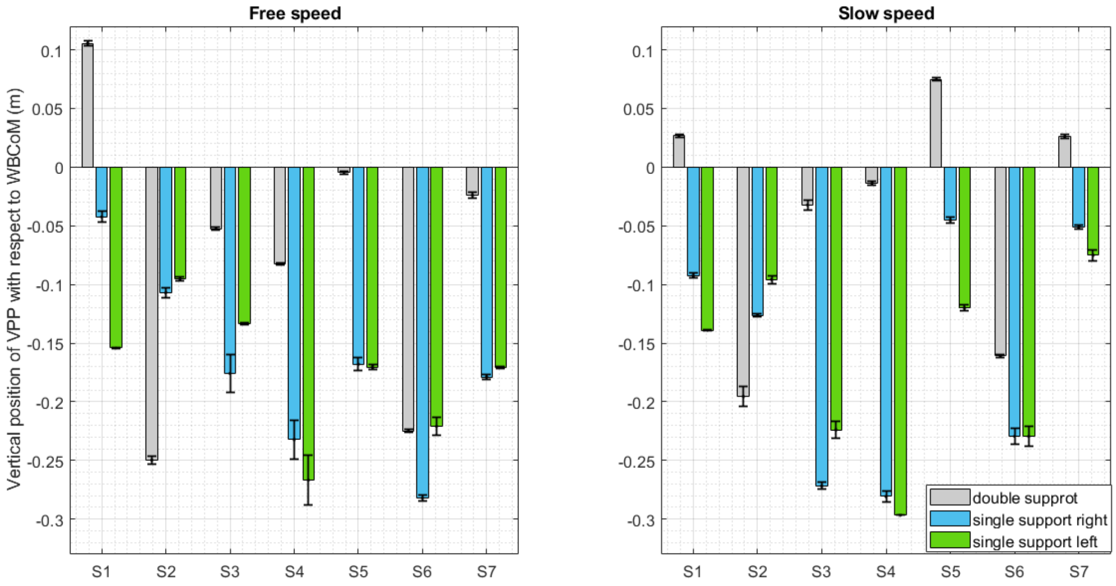


Fig. 5: The vertical position of VPP in the WBCoM-centered coordinate frame for different subjects (see Tab. I) at free and slow speeds. The height of the bars represents the mean of vertical distance of VPPs from WBCoM for each subject in several trials. The error bar indicate variance of VPP's height.

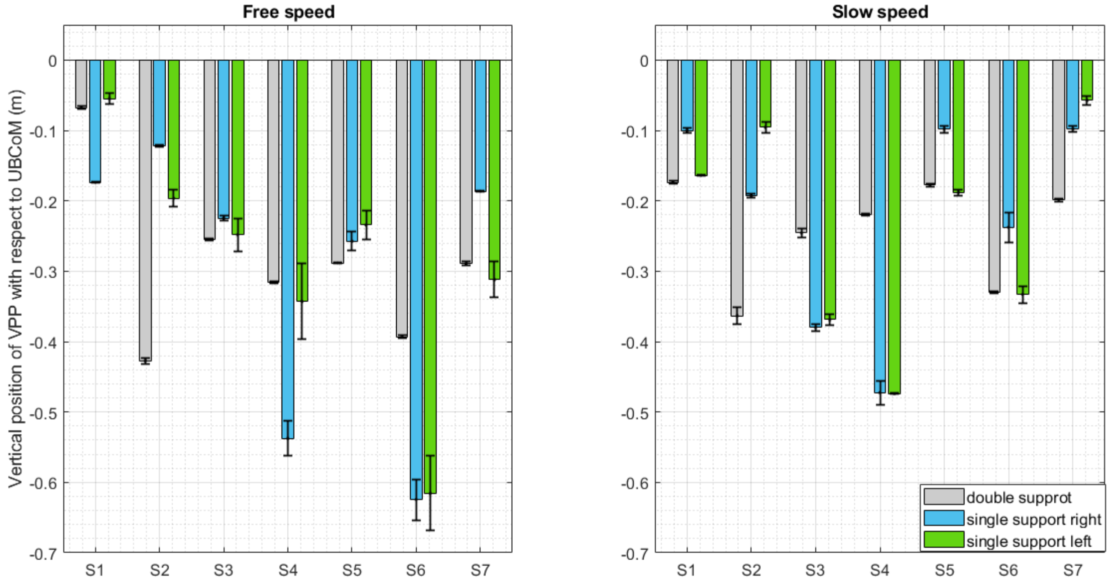
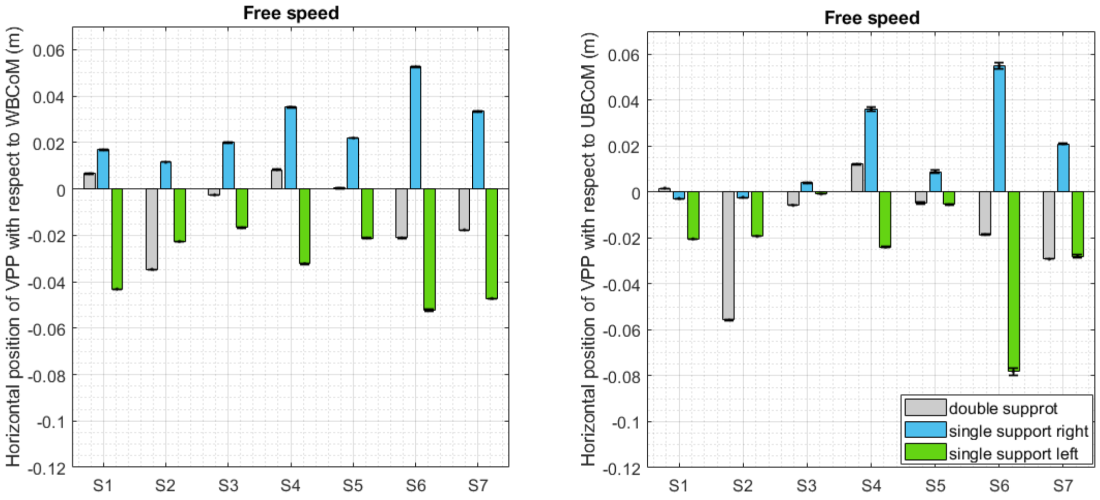


Fig. 6: The vertical position of VPP in the UBCoM-centered coordinate frame for different subjects (see Tab. I) at free and slow speeds. The height of the bars represents the mean of vertical distance of VPPs from UBCoM for each subject in several trials. The error bar indicate variance of VPP's height.

IV. DISCUSSION

For analyzing lateral posture control, VPP method is selected in this paper, which was not investigated before. The first finding is that VPP is clearly visible in all experimental trials for coordinate systems centered at both whole body CoM (WBCoM) and upper body CoM (UBCoM). Unlike VPP in sagittal plane, here, VPPs are mostly placed below CoM which mimics inverted pendulum model. On one hand, we found that in the UBCoM-centered reference frame, the variance of VPP's position is more than that of WBCoM-centered reference frame. On the other hand, the VPP vertical positions are more consistent in the UBCoM-centered frame.



(a)

(b)

Fig. 7: The horizontal position of VPP for different subjects (see Tab. I) at free speed. The height of the bars represents the mean of horizontal distance of VPPs from WBCoM for each subject in several trials. The error bar indicate variance of VPP's horizontal displacement. a) WBCoM-centered coordinate frame, b) UBCoM-centered coordinate frame.

To this end, it is difficult to conclude if TIP model is useful for upper body or whole body posture control. In addition, it is fair to say that balancing one of these systems will effectively address balance control of the other system.

Since negative VPP height is the most consistent finding in all sub-phases of walking and at different speeds, first, we describe how such a hybrid system can generate stable behavior. We found that in the right leg single support phase (RSS), the VPP is mainly located at the right side of CoM and in left single support phase (LSS), it is placed at the left side of CoM (see Fig.7). As shown in Fig. 8a, this model results in falling of CoM to the left side. With the touch down of the left leg, the pivot point moves to the left side of the CoM. Although switching the pivot point decelerate the pendulum movement the initial horizontal speed of the CoM (to the left) is sufficient for the second inverted pendulum (in the DS phase) to continue its leftward movement and to pass the vertical alignment. Then, the motion to the left speeds up until takeoff of the right leg which switches the pivot point for the second time and results in LSS phase. This speed is not sufficient to pass vertical alignment of the inverted pendulum and it moves back and forth (left and right) to return to its LSS initiation position. The reverse motion in DS continues to reach the RSS phase and this loop holds during walking.

This *TIP model* can successfully predict human lateral balancing (see Fig. 3 and Fig. 4). Interestingly, destabilizing movement of the VPP-CoM pendulum in the associated coordinate frame is not equal to instability of the whole body or upper body posture in the world coordinate frame as was explained in Sec. II-B and Fig. 2. Therefore, this TIP model that shows an energy efficient lateral balancing approach can be also used as a control method. By implementing VPP within FMCH [17] framework this method can be translated to have different adjustable compliant elements between upper body and the legs which switches the resulting VPP points in different phases. In [28], a 3D FMCH model is developed for predicting healthy and impaired walking in 3D. With here presented TIP model, the introduced constraints of [28] can be released.

Fig. 8b present another lateral balance model in which the VPP of DS is placed above CoM. The description of this model functionality in single supports are similar to that of TIP model while the order of acceleration and deceleration in DS will differ. The motion behavior predicted by this control strategy is less frequent in our experimental analyses.

Stability of a hybrid (switching) system which is composed of unstable subsystem is a well-known concept in control engineering literature [29], [30]. Changing switching conditions (e.g., time or surface) is one way for control of such hybrid systems [31]. As walking is laterally unstable and requires active feedback control for stability [13], [32], the findings of our study supports the idea of controlling switching conditions using the position of VPP. It means that humans use active control at the toe-off and heel-strike to change VPP's position. This discrete version of controlling inverted pendulum system can be interpreted as an extension of impulsive torque control for human standing [33] to a more complex space.

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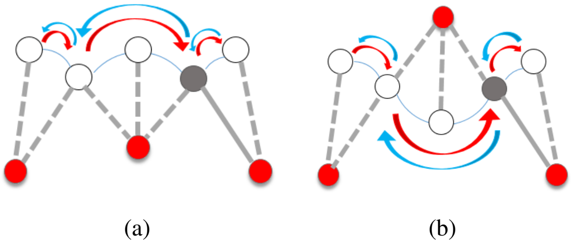
(a) (b)

Fig. 8: Proposed pendulum-based model of lateral posture control with sequential switching between a) inverted pendulum (IP) in the *TIP model* and b) combination of regular and inverted pendulum models.

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