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Compliant ankle function results in landing-take off asymmetry in legged locomotion

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AUTHOR-HIGHLIGHTS

- An extended SLIP model with a compliant ankle joint and a rigid foot segment is proposed.
- Proposed model represents extended foot contact in human running.
- Landing-take off asymmetry in legged locomotion is caused by asymmetric lever arms.

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ABSTRACT

The spring loaded inverted pendulum (SLIP) model is widely used to predict and explain basic characteristics of human walking and running. Its periodic running solutions can be mirrored at the instant of the vertical orientation of the leg and thus are symmetric between landing and take-off. In contrast, human running shows asymmetries between touchdown and take-off (e.g. shorter brake than push duration, greater mean ground reaction force during braking phase). Yet it is not fully understood whether these asymmetries are caused by asymmetric muscle properties (e.g. velocity-dependent force generation) or the asymmetric lever arm system in the human leg. We extend the SLIP model by a foot segment and a compliant ankle joint. This represents the extended foot contact and the displacement of the center of pressure during contact. With this model we investigate to which extent the landing-take off asymmetry in legged locomotion is caused by this asymmetric lever arm system. We find similar landing-take off asymmetries as in human running suggesting that the asymmetric lever arm system contributes to the asymmetry.

1. Introduction

To describe the basic dynamics of human locomotion template models (Full and Koditschek, 1999), the spring loaded inverted pendulum (SLIP) model is commonly used (Blickhan, 1989; McMahon and Cheng, 1990; Geyer et al., 2006). The SLIP model predicts basic characteristics of human walking and running like the gait-specific pattern of the ground reaction force (GRF) and the center of mass (COM) trajectory.

Periodic running solutions of the SLIP model typically show a symmetry between touchdown and take-off in GRF and vertical COM trajectory. In contrast, human running shows an asymmetric touchdown-take-off behaviour (Cavagna, 2006; Cavagna and Legramandi, 2009) with a higher take-off than touchdown height. While it is not fully understood, whether the asymmetric lever system of the limb or the velocity-dependent muscle properties are the reason for the asymmetry in human running, new experiments (Cavagna et al., 2011) indicate that asymmetric lever function is highly responsible for the asymmetry in leg function. The asymmetry in leg function for example is reflected in a shorter brake duration (negative external work during stance) than the push duration (positive external work during stance) and a greater mean ground reaction force during the braking phase than the mean ground reaction force during push off phase (Cavagna and Legramandi, 2009). In backward running the asymmetry is reversed (i.e. longer braking than push off) indicating the asymmetry in lever arms as the main reason for the asymmetry in leg function (Cavagna et al., 2011).

As asymmetric limb posture is prominently represented in the human foot function, we aim at understanding the effect of such a foot segment. Due to the plantigrade limb posture, humans can roll over the entire foot during contact. Then, the ground contact is initiated with the heel or the middle part of the foot and then contact shifts (Lieberman et al., 2010; Cavanagh and Lafortune, 1980). The center of pressure (COP) is moving from heel to toe

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until heel-off during contact (Murray et al., 1967). The distance the COP moves during contact is approximately 23% leg length in walking and 18% leg length in running (Lee and Farley, 1998). The motion of the COP in walking is similar to a rolling wheel with a radius of approximately 15% body height (Hansen et al., 2004) or 30% leg length (McGeer, 1990). Bullimore and Burn (2006) proposed a SLIP model with a prescribed, time-dependent linear displacement of the foot point to determine the mechanical consequences of the displacement of the COP. While the basic running dynamics remained similar, the displacement of COP led to lower peak horizontal GRF and reduced mechanical work per step. Also the periodic running solutions still exhibited symmetric contact phases.

Here we ask, how an elastic ankle joint will affect leg function during stance phase in running. We hope to explain to which extent an elastically coupled foot segment affects the COP displacement and thus contributes to the experimentally observed landing-take-off asymmetry during running. In order to analyze this asymmetry we extended the SLIP model and compared the results predicted by the model with the data obtained from human running. With this additional foot segment we expect to represent the asymmetric leg function and the corresponding COM movement in running more realistically. More specifically we ask, whether an elastically coupled foot segment can explain the observed COP displacements during the stance phase of human running.

2. Methods

2.1. FSLIP model

The foot spring (FSLIP) model is based on the SLIP model (Blickhan, 1989; McMahon and Cheng, 1990). The center of mass (COM) is described by a point mass which is attached to a prismatic leg spring. The foot is modelled as a massless rigid segment and coupled to the leg via a torsional spring (Fig. 1). The movement is restricted to the sagittal plane. The telescopic leg spring produces forces proportional to the difference between resting length L_0 and actual length L corresponding to the leg stiffness k. Similarly, the ankle spring creates torques proportional to the excursion from the resting angle φ_0 to the actual angle φ depending on the ankle joint stiffness c. The resulting leg forces in x- and y-direction (F_x and F_y respectively) are as follows (see Fig. 1 for the definitions of the geometrical properties):

$$F_x = k(L_0 - L)\cos\varphi_2 + \frac{c}{L}(\varphi_0 - \varphi)\cos\left(\varphi_2 + \frac{\pi}{2}\right) \quad (1)$$

$$F_y = k(L_0 - L)\sin\varphi_2 + \frac{c}{L}(\varphi_0 - \varphi)\sin\left(\varphi_2 + \frac{\pi}{2}\right) - mg \quad (2)$$

$$L = \sqrt{(x - f\cos\varphi_1)^2 + (y - f\sin\varphi_1)^2} \quad (3)$$

with the gravitational acceleration g and the mass m.

The forces created by the leg spring and ankle joint spring as well as the gravitational force are the only forces acting on the COM. No additional forces or changes in the impulse of the COM (for example at touchdown or take-off) are considered.

2.2. Periodic solutions

In running, support phases alternate with flight phases. As the swing leg dynamics are not considered here, it is sufficient to model just one leg. During contact leg forces are caused by compression of the prismatic leg spring (F) and by the torsional ankle spring (F_τ). Take-off occurs when the leg force decreases to zero. During flight the leg is oriented with a constant angle of attack (α_0) in preparation of the next touchdown (Seyfarth et al., 2002). The landing velocity of the COM and the leg angle determines the foot velocity at touch-down. In our simulations, the leg angle is assumed to be fixed during flight phase, i.e. foot velocity is equal to COM velocity in this phase. The resting angle of the ankle joint is chosen such that the heel strikes first. With the help of Poincaré return maps we search for periodic running solutions (Seyfarth et al., 2002; Rummel et al., 2010). As the model is conservative, the energy of the system remains constant. At apex, the system state is then uniquely characterized by the COM height. Periodic running solutions are identified by two equal subsequent apex heights.

2.3. Experimental data and model parameters

We used an experimental data set of six subjects (50 contacts per subject) running on a treadmill at 3 m/s (for subject information see Table 1). Ground reaction forces (GRF) separated for each leg were measured with piezo-electric force transducers integrated in the treadmill at a frequency of 1000 Hz (for more details see Lipfert, 2010). The marker data from anatomical landmarks and the three-dimensional ground reaction forces were corrected for synchrony (Günther et al., 2005; Lipfert et al., 2009; Maus et al., 2011). The initial COM position in sagittal plane was computed using anthropometric data (Dempster, 1955; Winter, 2009). The COM trajectory was estimated by integrating the ground reaction forces twice (Cavagna, 1975). The data was used to show the GRF and COM averaged over 50 steps of one exemplary subject (Fig. 2) and the mean force–length-curve averaged over all subjects and steps (Fig. 4).

The model parameters were chosen to be in the range of human running data (Günther and Blickhan, 2002; Blum et al., 2009) and result in an asymmetry in leg function as found experimentally. The parameters were chosen to be

$m = 80$ kg
$L_0 = 1$ m
$f = 0.25$ m
$k = 20$ k N m
$\alpha_0 = 74.5^\circ$

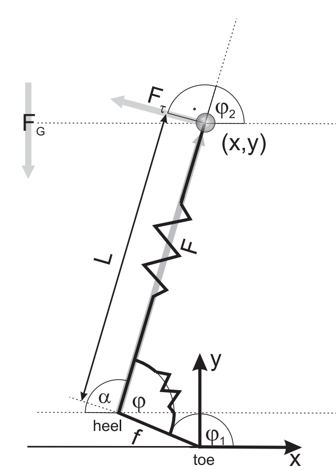

Fig. 1. Geometric properties of the FSLIP model.

Table 1
Subject characteristics.

N	Body mass (kg)	Body height (m)	Age (yrs)
6	77.5 ± 8.8	1.8 ± 0.1	23.7 ± 1.1

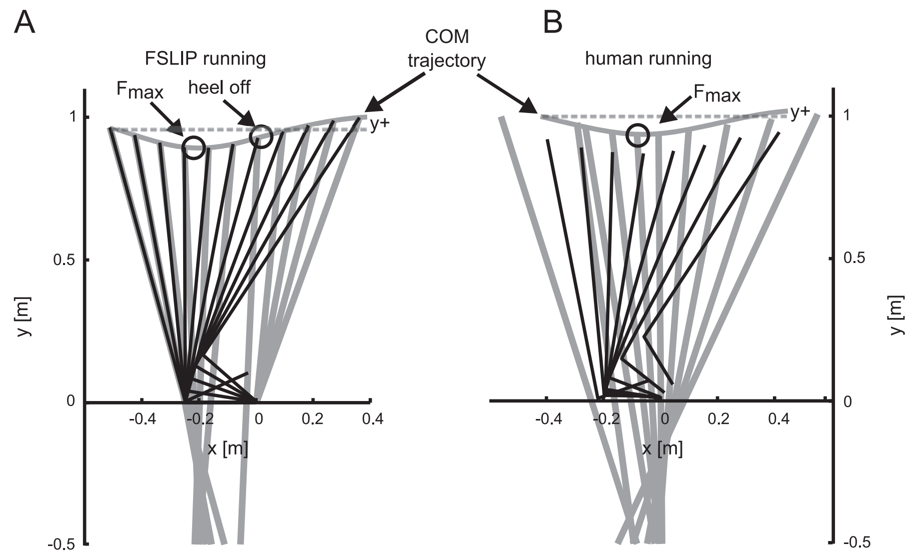

Fig. 2. Stance phase (A) in FSLIP and (B) in human running. Black lines show the leg spring axis and the foot segment (FSLIP model) respectively the line from hip to heel and heel to metatarsal marker (human running) and grey lines indicate the alignment of the GRF. The normalized ankle joint stiffness in the FSLIP model (A) is $c' = 1$, the angle of attack is $\alpha_0 = 74.5^\circ$ and the resting angle 100°. In this periodic solution the apex height is $y_{apex} = 1$ m and the horizontal velocity at apex is $\dot{x}_{apex} = 3$ m/s. The experimental data (B) shows the results from one subject averaged over 50 steps.

$$c' = \frac{c}{\frac{1}{2} mgL_0} = 1$$

The ankle joint stiffness c can be normalized to the resting leg length L_0, mass m and the gravitational acceleration. Normalizing the experimentally observed ankle joint stiffness reveals values between $c' = 1.1$ and $c' = 1.4$ (Günther and Blickhan, 2002). We choose the normalized ankle joint stiffness in the FSLIP model to be $c' = 1$ as it is close to the experimental data and it shows the asymmetry in the FSLIP model most dominantly.

2.4. Definition of center of pressure

The center of pressure (COP) is defined as the intersecting point of GRF with ground (Murray et al., 1967). The COP in the FSLIP model is obtained by calculating the force caused by the leg spring and ankle joint spring. The GRF is the sum of the translational leg spring force and a perpendicular force created by the ankle joint torque (rotational spring). As the GRF points to COM (point mass), the COP is defined by the intersection of the GRF with ground. After heel-off, the COP is fixed at the foot tip. In this phase (heel-off to take-off), the model has an additional degree of freedom, i.e. the position of the heel has to be determined. With the given position of the COM the foot segment can rotate around the foot tip. Different foot segment angles (φ_1, see Fig. 1) lead to different energies stored in the leg spring and ankle joint spring. The foot segment angle (and the corresponding heel position) which results in the smallest sum of energy stored in the leg spring and ankle joint spring is taken to determine the position of the heel. We define the effective leg as the vector pointing from the COP to the COM. With this definition a force–length-relation can be derived showing the combined effective stiffness of the leg and ankle joint spring during stance phase.

3. Results

3.1. Periodic running solutions

A representative solution with the apex height $y_{apex} = 1$ m and the horizontal velocity $\dot{x}_{apex} = 3$ m/s of a periodic pattern predicted by the FSLIP model is shown in Fig. 2A.

After ground contact is initiated with the heel the COM descends and the leg spring as well as the ankle joint spring are being compressed until maximum leg compression. Then the leg starts to extend and the COM is lifted again. During contact the GRF intersects at a point underneath the foot. Afterwards (around $\frac{2}{3}$ of contact time) the heel lifts off and the heel rotates around the foot tip. At take-off the COM is higher than at touchdown height.

3.2. Asymmetry in leg function

When the ankle joint spring is compressing the COP shifts from the heel to the foot tip. Once the COP reaches the foot tip, the heel lifts the ground and the COP remains at the foot tip until take off. The shift of the COP in the here described FSLIP model is not prescribed (e.g. shift with constant speed as in Bullimore and Burn, 2006) but a result of the underlying model structure and foot placement strategy.

The maximum GRF is reached at around 36% of contact time (Fig. 3A). At 66% contact time the GRF shows a change in slope (kink) followed by a shallower descent in GRF. The braking phase (i.e. the phase where negative work is performed on the COM during stance) is shorter than the push-off phase (positive work performed on COM during stance). Fig. 3B shows the leg spring and the ankle joint spring power in the FSLIP model. Compared to the positive (and negative) peak in the leg spring power the peak in the ankle joint spring power is delayed.

While the heel is in contact with the ground the effective stiffness is higher than after heel off (Fig. 4A). The drop in leg stiffness is counteracted by an increase in effective leg length.

3.3. Force–length-relation

Based on the leg definition in Section 2.4, the force–length-curves predicted by the FSLIP model (Fig. 4A) and from experimental data (Fig. 4B) can be compared. These curves show a change in leg stiffness during stance. During middle foot contact the leg spring can compress independently from the ankle spring. As long as the foot is flat on the ground, ankle flexion is constrained by heel contact (phase 1, Fig. 4). After maximum leg compression the ankle spring is still compressing while the leg spring already extends. The result is a drop in leg force with little

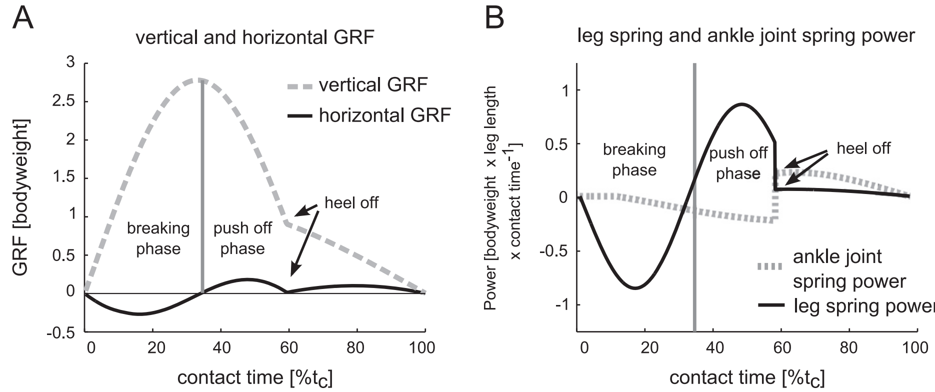

Fig. 3. (A) Vertical and horizontal ground reaction forces in a periodic running solution predicted by the FSLIP model. The arrows indicates heel-off. The grey vertical line separates the braking from the push-off phase. (B) Leg spring and ankle joint spring power in a periodic running solution predicted by the FSLIP model. The peak in ankle joint spring power is delayed compared to the leg spring power. The normalized ankle joint stiffness was chosen to be 1 and the resting angle was chosen to be 100°. In this periodic solution the apex height is $y_{apex} = 1$ m and the horizontal velocity at apex is $\dot{x}_{apex} = 3$ m/s.

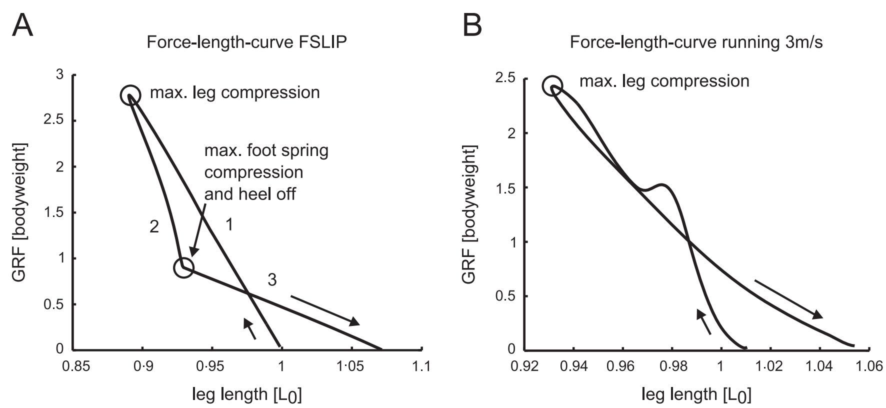

Fig. 4. Force–length-curve of the stance leg in FSLIP (A) and experimental data averaged over all subjects and steps (B). The arrows indicate the time-course of the force–length-curve. The normalized ankle joint stiffness in the FSLIP model was chosen to be 1 and the resting angle was chosen to be 100°. In this periodic solution the apex height is $y_{apex} = 1$ m and the horizontal velocity at apex is $\dot{x}_{apex} = 3$ m/s.

change in leg length (phase 2, Fig. 4). During heel-off phase the one-sided constraint at the ankle joint is released and the two springs extend in series which results in a decreased overall leg stiffness and a flatter slope of the force–length curve (phase 3, Fig. 4). The resulting drop in leg stiffness is associated with an increase in effective leg length. The here shown combination of a linear and a torsional spring can modify leg stiffness and rest length during contact depending on the selected foot placement strategy. The force–length-curves predicted by the FSLIP model and the experimental results show both a counter-clockwise loop with a higher stiffness during leg compression than during leg extension. A striking difference between model prediction and experimental data is the lack of the impact peak in the FSLIP model.

4. Discussion

4.1. Asymmetry in leg function

In this study we investigated how a compliantly attached foot segment affects the dynamics of running as predicted by the FSLIP model.

The FSLIP model inherits basic properties of the underlying SLIP model. Especially the shape and size of the region of stable solutions remain almost unchanged (Maykranz et al., 2009). The most striking difference to the SLIP model is the introduction of an asymmetry between touchdown and take-off.

The observed running solutions are characterized by a higher take-off than touchdown height. This lift of the COM during stance (y^+ in Fig. 2) is also found in experiments (Cavagna, 2006) but is even more pronounced in the model. One reason could be the missing metatarsal joint in the FSLIP model which does represent the bending of the toes and thus increases the lever arm of the foot segment. This lift of COM during stance is achieved even though the model is energy conservative and is associated by an increasing leg length during stance.

As the effects of increased leg length and reduced leg stiffness during leg extension are derived based on an energy conservative model, these two effects as displayed in the force–length-curve do not correctly describe the work done on the COM. Typically the area under a force–length-curve is interpreted as the work done associated with the movement and described as a work-loop (Josephson, 1985). In more-dimensional systems this is generally not the case. In a conservative system the work done on the COM should equalize to zero after one cycle, otherwise the body would change its energy. Within the FSLIP model the area enclosed in the force–length-curve does not equalize to zero. In this case the work done on the COM depends on the change in leg length and the shift of the COP. This work is represented by the enclosed area under the force–length-curve plus the work done required to move COP (Maykranz et al., 2013). As this model is conservative, there is no external work provided for the displacement of the COP. Instead, it requires an internal shift of energy. The required work to move the COP is a consequence of the used leg definition.

Although the FSLIP model consists of ideal elastic springs with a perfect elastic recoil, the asymmetry in lever arms result in asymmetric GRF with a shorter brake phase and a longer push off phase (Fig. 3) like found in human running (Cavagna, 2006). Also, the mean GRF during push is lower than during braking. The FSLIP model therefore shows the same asymmetries as found in human running without energy losses or active muscle work as suggested previously. Together with the reversed asymmetry observed in human backward running (Cavagna et al., 2011), this indicates that the asymmetric lever arms created by the foot can cause the observed landing-take-off asymmetry in human running. If the muscle properties would be the dominant reason and the geometry of the leg segments would contribute much less to the asymmetry, then in backward running the asymmetry of GRF would just be smaller instead of being reversed.

The ankle joint stiffness in the FSLIP model is chosen to be close to human data and resulted in the here presented asymmetries. However, the asymmetries may revert with a large change in parameters. Up to a normalized ankle joint stiffness of $c' = 3.0$, the here presented asymmetries in braking and push-off as well as higher take-off than touchdown are found. With a normalized ankle joint stiffness greater than 3.0, the asymmetry in push-off and braking time gets reversed. In contrast, the asymmetry in touchdown and take-off height gets smaller but does not reverse. For very large ankle joint stiffness the FSLIP model merges into the SLIP model and the asymmetries vanish.

4.2. *Center of pressure (COP)*

In the FSLIP model the center of pressure displacement during contact is a result of the underlying structure instead of a predefined displacement with constant speed (Bullimore and Burn, 2006). In contrast to the linear point of force translation (POFT) (Bullimore and Burn, 2006) the displacement here shows a nonlinear time dependency which reflects the asymmetry between leg compression and leg extension. As soon as the COP reaches the foot tip, the COP remains constant while the COM rotates around that point.

The human displacement of the COP is also non-linear in time but shows several phases including initial contact phase, forefoot contact phase, foot flat phase and forefoot push off phase (De Cock et al., 2008). These conceptual phases of the displacement of the COP found in human locomotion cannot be fully explained with this model. To replicate these phases in more detail, additional joints to simulate the metatarsal joints or the knee joint would be required. In the FSLIP model, the excursion of the COP abruptly stops at heel-off. The heel-off in humans is smoother due to elastic properties in the foot segment and a sequential lift-off due to several joints in the foot. The late heel-off predicted by the FSLIP model (66% of contact time) is also because of a sequential lifting of heel and ball of the foot before toe-off (De Cock et al., 2005)). The heel-off timing in the model depends on the ratio between leg and ankle joint stiffness. A higher ankle joint stiffness would result in an earlier heel-off.

Before heel-off, the COP moves from the heel to the foot tip. In this phase, the GRF intersects in a point underneath the ground creating a virtual rotating point. The leg behaves as if rotated around this point which causes an increased effective leg length. The rotation of the leg is accompanied with the shortening and lengthening of the leg as the COM does not follow a circular path. This virtual pivot point is not fixed but moves in forward direction. As soon as the COP reaches the foot tip the heel lifts off and the forces intersect at the foot tip. Experimental results also show that the axis of GRF intersect underneath the ground. This intersection does not take place at an explicit point but rather in a confined area. The observed experimental virtual pivot point lies further in anterior direction. This may be caused by the toes creating an additional foot segment which is not covered in the model. With an increased leg length, the COM trajectory can be smoothed by having less vertical excursions and thus leading to less vertical external work. This was already predicted by the POFT model (Bullimore and Burn, 2006).

4.3. *Leg simplification*

The FSLIP model demonstrates that asymmetric leg function as observed in animal and human locomotion may be largely shaped by the arrangement of the leg segments. In human locomotion, the flat contact of the foot introduces a phase, where ankle flexion is constrained by heel contact, i.e. the heel cannot move further downward. As a result, energy storage and release at ankle joint may be timed differently to the energy storage and release at the leg axis. This leads to systematic changes in leg function during contact such as decrease in leg stiffness and increase in leg length until take-off. A more profound understanding of this mechanisms could help to explain the gait-specific operation of the segmented leg in human walking and running. In this context, the elastically coupled foot segment appears to play a key role, as demonstrated with the FSLIP model.

The here presented FSLIP model conceptualizes the function of the human leg by an arrangement of two springs. The thigh and shank are abstracted with a prismatic leg spring. The foot segment is coupled to this prismatic leg by a torsional ankle spring. This simplification has the advantage that the leg can be described with just a few parameters. Interestingly this relatively simple leg shows no constant leg stiffness during the stance phase. The assumption of a constant leg stiffness for the entire stance phase is also challenging in human running. Depending on the used method deviations of more than 50% in the determination of leg stiffness can occur (Blum et al., 2009). The shift of COP due to foot contact makes it difficult to determine stiffness on global leg level. The here presented FSLIP model illustrates the problem of establishing a proper leg definition which is valid and reasonable for the entire stance phase. Depending on the model complexity leg definitions based on anatomical marker sets are not always applicable because anatomical landmarks may not be included in the model. The here chosen leg definition as a vector pointing from COP to COM is a generalization which is independent from the model complexity. Thus this generalized leg definition is suitable to compare model and experimental data even though this definition describes a functional leg instead of the anatomical leg. Additionally, this definition provides a simple approach to determine the work done on the COM by the overall leg.

Shortcomings of the simplified leg of the FSLIP model are the lack of a knee joint and the missing of biarticular structures. Biarticular muscles spanning knee and ankle joint could play an important role for synchronization and coupling between joints (Jacobs et al., 1996; Novacheck, 1998). For steady, level gaits the energy of the body needs to be approximately constant. This is not the case for individual joints, e.g. the ankle provides net positive work while the knee provides negative work (Novacheck, 1998). The elastic structures in the FSLIP model are conservative individually and thus cannot provide a net positive or negative work. Despite these shortcomings, the FSLIP model shows that asymmetric touch-down take-off leg function as described e.g. by Cavagna (2010) could also be caused by the asymmetric leg design with the foot pointing forward. A three segmented leg model with a rotational knee joint spring in addition to the here presented ankle joint spring could improve the predictions of the FSLIP model regarding the early heel off. The heel-off timing is sensitive to the ratio of ankle joint and knee joint stiffness. With an elastic coupling between these two joints (similar to the human gastrocnemius muscle) the stiffness ratio may not need to be perfectly matched to achieve a desired heel-off timing.

4.4. Conservation principles

Conservation laws are fundamental principles in physics that can help in the evaluation of experimental data. For example, if running with constant speed, the COM energy at every instant during the flight phase is constant. Deviations in COM energy during this phase should be critically questioned. Significant intra-individual stride variabilities are observed (Belli et al., 1995) but these should cancel out if averaged over a sufficient number of steps .

The here presented model FSLIP is energy conservative. The symmetric SLIP model additionally has the property that the momentum lost during brake is equal to the momentum gained during push. The asymmetric FSLIP model does not fulfil this conservation of momentum during contact. With the take-off height in the foot-spring model being higher than the touchdown height an increase in potential energy between these instants occurs. As the model is energy conservative this results in a decreased kinetic energy at take off. Consequently the momentum at take-off is smaller than during touchdown. As human running also shows a higher take-off compared to touchdown height it is arguable, whether conservation of translational momentum can be assumed in human running (Cavagna, 2006).

Given conservation of energy, the amount of work done during the braking phase needs to be equal (but opposite in sign) to the work done during push:

$$W_{brake} = -W_{push} \tag{4}$$

As work divided by time is the mean power, this equation can also be displayed as mean power multiplied by time. With this relationship it becomes clear that the mean power during each phase is inversely proportional to the phase time:

$$\overline{P}_{brake} t_{brake} = -\overline{P}_{push} t_{push} \tag{5}$$

Hence, the advantages gained through the asymmetry, i.e. lift of COM during stance, increased virtual leg length, requires an increased mean power during the braking phase. This is reflected in the function of muscles, which are able to generate more power during negative work (eccentric operation) than during positive work (concentric operation). At the same time, the asymmetry introduced by the geometry and elastic coupling of the foot prolongs the push-off phase and thus increases the ability of the leg muscles to perform positive work (Cavagna, 2010).

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