

Chapter 2

Fundamental Subfunctions of Locomotion

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Legged locomotion is a complex hybrid, nonlinear and highly dynamic problem. Animals have solved this complex problem as they are able to generate energy efficient and robust locomotion resulted from million years of evolution. However, different aspects of locomotion in biological legged systems such as mechanical design, actuation and control are still not fully understood. Splitting such a complicated problem to simpler subproblems may facilitate understanding and control of legged locomotion. Inspired from template models explaining biological locomotory systems and legged robots, we define three basic locomotor subfunctions: stance leg function, leg swinging and balancing. Combinations of these three subfunctions can generate different gaits with diverse properties. Basic analysis on human locomotion using conceptual models can result in developing new methods in design and control of legged systems like humanoid robots and assistive devices.

PREAMBLE—THINGS TO CONSIDER "BEFORE WALKING"

Animals are integrated collections of “parts” that are good enough to be passed on to the next generation. Because we often project a forward design philosophy on animals, it is all too common to misinterpret animals as optimal designs that should be copied. To the contrary, animals are junkbots that inherit “parts”—really a combination of genetic and epigenetic factors—from their ancestors. Of course, animals’ structures can be modified from generation to generation but the evolutionary process is more akin to gathering parts from old VCRs and printers than to the forward-design process used in engineering. Animals build bodies that are good enough for survival in their environment, for competition with other individuals of the same species, and for reproduction to pass their genes on to the next generation—this is the key tenet of evolution by natural selection. Additionally, there are limitations to the body types, shapes, and sizes that a given animal lineage can build. The form and function of an animal’s

body is subject to developmental, material (i.e., biochemical and tissue-level), and constructional constraints that are inherited from their ancestors.

When considering bio-inspired designs for robots, it is important to look at animals for what they are and how they have been built from the biomaterials available to them. Despite this caveat, nature has produced excellent designs and a level of performance in legged locomotion that is unrivaled by robotic systems. Hence, a guiding principle for robot builders might be to begin with models that capture the fundamental physics and control strategies rather than attempting to mimic the fleshy details of animal locomotion.

In the history of life on Earth, leg-like appendages appear to have evolved independently within many phyla, including Arthropoda, Annelida, Mollusca, Echinodermata, and Chordata (Panganiban et al., 1997). In every case of this messy history, legs originated in an aquatic environment and may have been used for swimming, underwater walking or bounding, grasping, feeding, or holding position in strong currents. The subsequent use of legs for locomotion on land introduced new challenges. Amongst functional changes in ventilation, respiration, desiccation resistance, temperature regulation, and metabolism, these terrestrial pioneers began to move without hydrodynamic forces—instead, using their legs to apply horizontal forces while supporting their full body weight without buoyancy. The transition to terrestrial legged locomotion has been accomplished by insects, arachnids, crustaceans, and vertebrates. A hypothesized role of the first vertebrate limbs were as hold-fasts used by our nearest known lobe-finned fish ancestor, *Tiktaalik*, in fast moving rivers and streams some 375 million years ago (Daeschler et al., 2006). The pelvic bones, femur, tibia, and fibula of *Tiktaalik* are homologous to our own, yet *Tiktaalik*'s appendages had the appearance of strut-like fins (Shubin et al., 2014).

Our ancestral hind limbs have a longer history as aquatic holdfasts than they do as bipedal walking or running legs. The early tetrapods that inherited the appendages of *Tiktaalik* used them to move about quadrupedally on land, as did the early amniotes and mammal-like reptiles. The first mammals used these limbs for quadrupedal terrestrial and arboreal locomotion some 200 million years ago, and the marmoset-like early primates committed to quadrupedal arboreal locomotion about 50 million years ago. Apes descended from quadrupedal primates and began a transition from quadrupedal arborealism to suspensory locomotion, wherein the forelimbs grasp and pull-up on branches while the hind limbs tend to grasp and press against branches, tree trunks, or the ground. The subfamily Ponginae, containing extant orangutans, gorillas, chimpanzees, and humans originated some 12 million years ago from an orangutan-like great ape. The first Hominin with features of habitual bipedalism, *Australopithecus afarensis* appeared in the fossil record about 4 million years ago and *Homo erectus*, with skeletal anatomy nearly the same as our own, appeared just 2 million years ago.

This necessarily stochastic evolutionary history finds us as the only obligate biped amongst primates—and the only striding biped amongst mammals. The only other obligate, striding bipeds living today are birds. Including their theropod ancestors, these avian dinosaurs have a 250 million year history as striding bipeds, compared with less than 5 million years in our own lineage.

The example of our own evolutionary history highlights the meandering path we have taken to bipedalism and the short evolutionary distance from our arboreal ancestors. Does their longer history of bipedal locomotion mean that birds are better “designed” or have a better handle on the physics of bipedalism? No—an organism at any given time is simply good enough to survive, compete, and reproduce in its ecological niche. Nonetheless, it is useful to consider the inherited design constraints and opportunities, as well as the different ecological niches of birds and humans. For example, natural selection has resulted in fast, economical running of ostriches and economical walking plus long-distance running capabilities of humans—yet these different specializations were shaped in part by the developmental, anatomical, physiological, and behavioral biology inherited from the ostrich and human ancestor. A primary goal of comparative biomechanics is to understand the fundamental physics of locomotion within an evolutionary context. As this understanding progresses, bio-inspired legged robots and robotic prosthetics will transition from copying nature to borrowing, in whole or part, its strategies for interacting with the physical world—thus matching or even exceeding the locomotor performance of biological systems.

Bearing these evolutionary caveats in mind, the remainder of this chapter seeks to reveal and interpret the fundamental physics of legged systems. Our approach divides legged locomotion into three subfunctions, which are intrinsically interrelated, yet represent distinct tasks:

- Stance (Chapter 2.1) is the subfunction that redirects the center of mass by exerting forces on the ground.

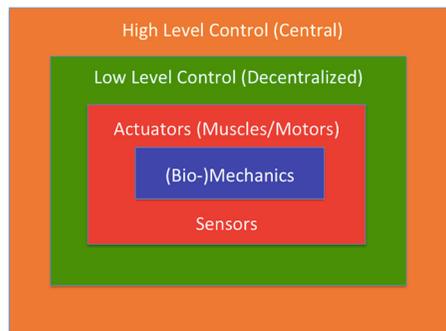


FIGURE 2.0.1 Functional levels of the locomotor system.

- Swing (Chapter 2.2) is the subfunction that cycles the legs between ground contacts.
- Balance (Chapter 2.3) is the subfunction that maintains angular velocity and body attitude within acceptable limits.

Legged locomotion is a complex task with integrated functional levels influencing all three locomotor subfunctions. These levels are mechanical, actuation, sensing, and both low- and high-level control of the animal or machine (Fig. 2.0.1). Our separate treatment of locomotor subfunctions allows interrogation of key functional features at each of these levels.

Chapter 2.1

Stance

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The stance subfunction of legged locomotion considers the ground reaction force exerted on the legs to redirect the body's center of mass from stride to stride. Stance is usually the first subfunction that comes to mind, perhaps because we, large mammals, spend quite a bit of time standing and walking and recognize that our legs keep the rest of our body elevated above the ground during these behaviors. On the other hand, small mammals (of body mass less than about 1 kg body mass), as well as lizards, crocodylians, and amphibians rest their bodies on the ground and tend to walk or run intermittently. From the perspective of these animals, the cycling of the legs from one stance to the next, the *swing* subfunction discussed in the following section might seem equally pervasive. The stance and swing subfunctions of the legs influence each other's dynamics and they also influence the third locomotor subfunction of this chapter, *balance*.

The primary function of the stance leg or legs is to interact with the ground and redirect the body during each stride of locomotion by imposing fluctuations in both the magnitude and direction of the force exerted on the ground. We often think of the stance leg as supporting the body against gravity, even to the extent that knee and ankle extensor muscle groups are sometimes called “antigravity” muscles. The summed vertical force of the stance legs in fact always oscillates about one body weight during steady-speed terrestrial locomotion, such that body weight can be thought of as an “offset.”

The stance legs use variations in force to redirect the center of mass, oscillating between braking and propulsive interactions with the ground. The net force in propulsive interactions is in the direction of travel, and that of braking interactions is against the direction of travel. Because of the geometry of the leg–substrate interaction, a forward (protracted) leg tends to cause braking and a backward (retracted) leg tends to cause propulsion.

In any analysis of stance, it is important to consider potential influences of gait, speed, animal size, leg number, and the dynamic interactions between the swing leg(s) and other body segments. The function of a given stance leg is linked to that of other simultaneous stance legs, as well as rotations and translations of leg and body segments. These latter considerations are important for animals and machines that have moving segments with real inertial properties, yet they are neglected in simplified point-mass models that only consider the total ground reaction force acting on the center of mass. Conversely, a stance leg can

influence swing and balance by exerting forces and yaw torques, plus roll and pitch torques if the foot is able to grasp or adhere to the substrate. With simultaneous contact of more than one limb, such as during double support of bipedal walking or during quadrupedal or multilegged gait, differential leg forces can produce force couples that also contribute to balance—for example, as the rear wheels of a car resist upward pitch during forward acceleration (Gray, 1968; Murphy and Raibert, 1985; Lee et al., 1999).

2.1.1 EFFECTS OF GAIT

In our treatment of *stance* as one of three locomotor subfunctions, the main objective of the stance legs is to redirect the center of mass, which can be achieved using any of several gaits. Gait is defined by a stereotyped spatiotemporal pattern of leg contacts and oscillations of the center of mass. Redirection of the center of mass may be achieved by using one leg at a time, as during bipedal running with aerial phases, or by using more than one leg at a time, as during all other gaits of bipeds, quadrupeds, and multilegged animals. The collective action of the stance leg or legs exerts oscillating vertical and shear forces to redirect the center of mass during each stride of locomotion.

Vertical center of mass oscillations are achieved using one or more leg at a time, but a given stance leg contributes to only one cycle of vertical oscillation per stride. In symmetrical gaits, which are defined by bilateral (left–right) limb pairs that are one-half stride cycle out of phase (Hildebrand, 1965), two cycles of vertical oscillation are achieved alternately by left and right legs of a pair during each stride. In asymmetrical gaits such as bounding, galloping, and bipedal hopping the collective action of the stance legs achieves a single cycle of vertical oscillation per stride. This achieves a single “gathered suspension” (a flight period with the legs folded under the body) in each stride. Exceptions to this rule for asymmetrical gaits are the fast gallop of cheetahs and greyhounds, as well as the half-bound of rabbits, which include both a “gathered” and “extended suspension”—representing two vertical oscillations per stride (Bertram and Gutmann, 2009). Hence, simultaneous leg forces produce two vertical oscillations per stride during symmetrical gaits and typically only one oscillation per stride during asymmetrical gaits. It is the norm for animals, including ourselves during walking, to exert locomotor forces simultaneously with more than one stance leg. Bipedal running, wherein only one stance leg exerts force at any given time, is the only exception to this rule. However, it may be argued that bipedal hopping of macropods and rodents also falls into this category, considering the two hind legs acting as one.

To affect vertical oscillations of the center of mass, the vertical force exerted by the stance leg or legs alternately rises above and then below body weight

during the stride. This is true of walking as well as running. In our bipedal running, vertical force rises above body weight during single-leg support and falls to zero during the aerial phase. Bipedal walking shows an opposite and somewhat counterintuitive pattern where vertical force rises above body weight during the double-leg support at the step-to-step transition and falls below body weight in the middle of single-leg support (Fig. 2.1.1A). Differences between bipedal walking and running can be illustrated by windowing the vertical acceleration in the middle of single-leg stance, showing a trough during walking and a peak during running (Fig. 2.1.1B). For example, comparing humans (black line) to guinea fowl (blue line) shows that guinea fowl use a running gait indicated by shallow peaks in vertical acceleration, whereas humans use a walking gait indicated by shallow troughs in vertical acceleration, at two of the intermediate speeds. Because center of mass position is given by the double integral of acceleration with respect to time, the center of mass reaches its lowest vertical position near maximum vertical acceleration—occurring in mid-stance of running and in double-leg support of walking. This difference between running and walking is the basis of a longstanding dichotomy emphasizing that the center of mass reaches its lowest position during mid-stance of running and its highest position during mid-stance of walking (Cavagna et al., 1976, 1977).

Stride dynamics can also be considered in terms of the kinetic and potential energy of the center of mass. Due to the braking impulse during the first half of leg contact, kinetic energy always reaches a minimum near the middle of single-leg stance during both running and walking of bipeds. Pairing this minimum in kinetic energy with the aforementioned potential energy minimum at mid-stance of running and the potential energy maximum at mid-stance of walking provided the impetus to advance two models to characterize running and walking: the spring loaded inverted pendulum (SLIP), a bouncing model with in-phase kinetic and potential energy for running; and a rigid inverted pendulum model with out-of-phase kinetic and potential energy for walking (Cavagna et al., 1977). These two mechanisms have long shaped our understanding of running and walking gaits in bipedal, quadrupedal, and multilegged animals (reviewed by Dickinson et al., 2000).

Bipedal running and hopping as well as quadrupedal or multilegged trotting are well described as “bouncing” gaits, defined by the spring-loaded inverted pendulum (SLIP) model (Blickhan, 1989; McMahon and Cheng, 1990). These gaits show maximum vertical force at mid-stance when the center of mass is at its lowest point, and may or may not include aerial periods between leg contacts. Because SLIP-like gaits may be achieved with leg springs, this provides a mechanism to reduce total energy cost by storing some of the energy elastically in the absorptive phase of early stance and returning it in the generative phase of late stance.

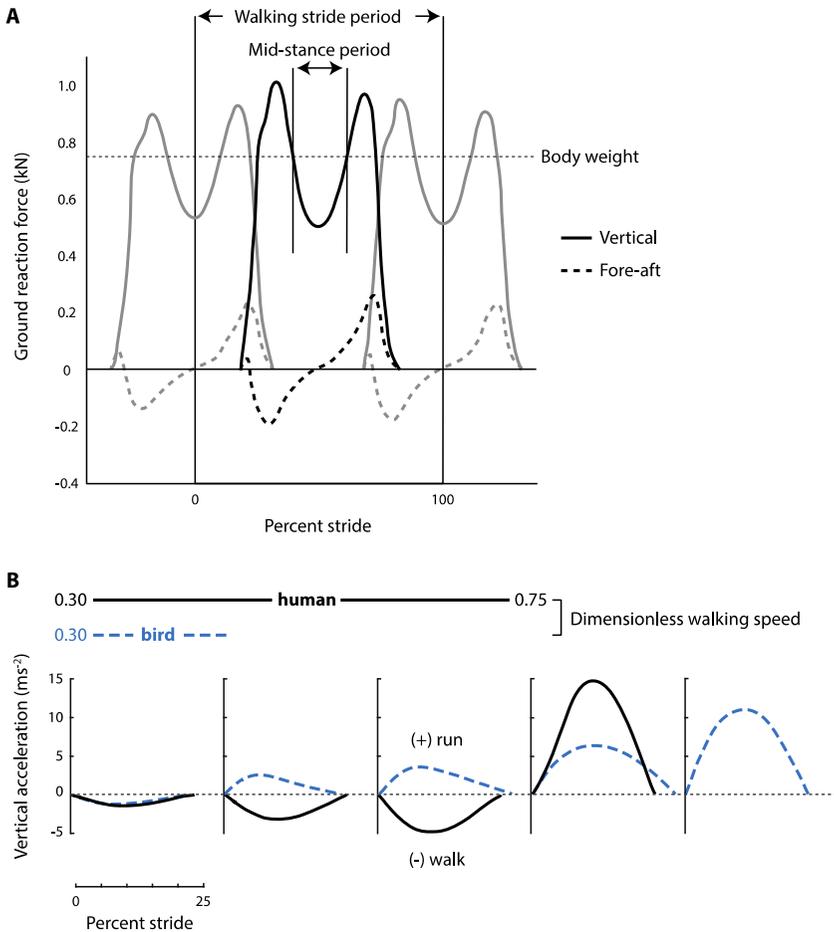


FIGURE 2.1.1 (A) Three human walking steps used to define a stride from mid-stance to mid-stance of the same limb (gray traces). Vertical ground reaction force is below body weight during the mid-stance period of walking. (B) Patterns of mid-stance vertical acceleration across a range of dimensionless walking and running speeds, including humans (black) and guinea fowl (blue). Negative mid-stance acceleration indicates walking and positive, running. Humans maintain negative accelerations up to dimensionless walking speeds as high as 0.75 but guinea fowl switch to running at dimensionless speeds corresponding to moderate human walking. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this chapter.)

Despite agreement of the bipedal running and quadrupedal or multilegged trotting gaits with the SLIP model, studies showed that bipedal and quadrupedal walking dynamics (e.g., [Lee and Farley, 1998](#); [Griffin et al., 2004](#), [Genin et al., 2010](#)) deviate substantially from a rigid inverted pendulum model. This is not unexpected given that the vertical force of a rigid inverted pendulum model

reaches a local maximum rather than the necessary minimum at mid-stance (Geyer et al., 2006). Likewise, vertical force will not reach the necessary maximum during the step-to-step transition unless double-leg support is modeled. Hence, it is difficult to reconcile the mechanics of a rigid inverted pendulum with the measured dynamics of walking. In contrast to the rigid inverted pendulum model, Geyer et al. (2006) also showed that simulations of walking on compliant legs can match force patterns observed during human walking, exhibiting a minimum vertical force at mid-stance and a maximum vertical force during double support of the step-to-step transition.

Gait dynamics can be better understood by considering the fundamental physics of the animal's interaction with the substrate. The center of mass reaches its lowest point during the transition between single-leg stances in walking, i.e., during double support at the step-to-step transition. Conversely, the center of mass reaches its lowest point during the middle of single-leg stance in running. Minimum center of mass height coincides with maximum vertical force in both gaits because this is where the center of mass is redirected from falling to rising. Hence, it might be argued that walking and running show similar vertical oscillations—simply achieved by two legs during walking and a single leg during running. However, walking and running in fact show fundamentally different dynamics. To understand what is driving these distinct dynamics, we need to consider the pattern of braking and propulsion during the downward to upward redirection of the center of mass. In SLIP-like bouncing gaits, the force on the center of mass is braking and then propulsive during the downward to upward redirection. Walking is the opposite: propulsive force precedes braking force during the downward to upward redirection. Thus, redirection of the center of mass pairs with opposite patterns of braking and propulsion in walking versus running. This observation provides the impetus for applying collision-based dynamics to legged locomotion.

The guiding principle of collision-based dynamics is that the stance leg or legs seeks to redirect the center of mass with the least mechanical work possible. Mechanical work can be viewed as an extension of D'Alembert's principle of orthogonal constraint, which holds that a force may redirect a mass with zero work as long as force and velocity vectors are kept perpendicular such that their dot-product, mechanical power, is zero (D'Alembert, 1743). Minimizing mechanical power at a given speed minimizes the animal's mechanical cost of transport. Collision-based costs are incurred whenever an animal's force and velocity vectors are not perpendicular. Thus, collision-free dynamics may be achieved only if the legged system is able to maintain a perpendicular relationship between the force and velocity vectors of its center of mass in every instance of stance (Fig. 2.1.2(right)) (Ruina et al., 2005; Lee et al., 2011). Because propulsion precedes braking during the step-to-step

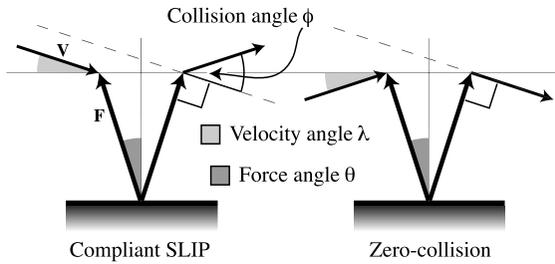


FIGURE 2.1.2 Collision-based dynamics for a SLIP model versus the zero-collision case (Lee et al., 2011).

transition of walking, the force vector can be kept more nearly perpendicular to the velocity vector while the center of mass is redirected from downward to upward, thereby reducing the mechanical work done by the stance legs on the center of mass (Fig. 2.1.2(right)). In contrast, SLIP-like bouncing dynamics cannot minimize the mechanical work required to redirect the center of mass because braking precedes propulsion during downward to upward redirection of the center of mass (Fig. 2.1.2(left)). This violates the principle of minimizing mechanical cost through orthogonal constraint, as seen in the zero-collision case. However, there is evidence in some species that part of the mechanical energy of the SLIP is stored in spring-like tendons that release elastic strain energy later in stance (Biewener, 2005).

Mechanical work is quantified by the mechanical cost of transport, CoT_{mech} , which is the work required to move a unit body weight a unit distance. CoT_{mech} can be determined from center of mass mechanical power—the dot-product of the force vector on the velocity vector. During level, steady-speed locomotion positive and negative work are equal in magnitude. Physiologist and modelers often count only positive mechanical power (e.g., Cavagna et al., 1977; Kuo, 2002), but here we take the absolute value of power to account for both positive (generative) and negative (absorptive) work:

$$CoT_{mech} = \frac{\sum |\mathbf{F} \cdot \mathbf{V}|}{n (mg \bar{v}_y)}, \quad (2.1.1)$$

where \bar{v}_y is the mean forward velocity, g is gravitational acceleration, m is body mass, and n is the number of time-intervals in the summation. From the perspective of a point-mass model (i.e., a model concentrating all of the system's distributed masses at the center of mass and considering only translations), SLIP-like bouncing gaits incur a much greater mechanical cost of transport than gaits, such as walking, that minimize collision-based costs. In humans, for example, the mechanical cost of transport during SLIP-like running is three-times

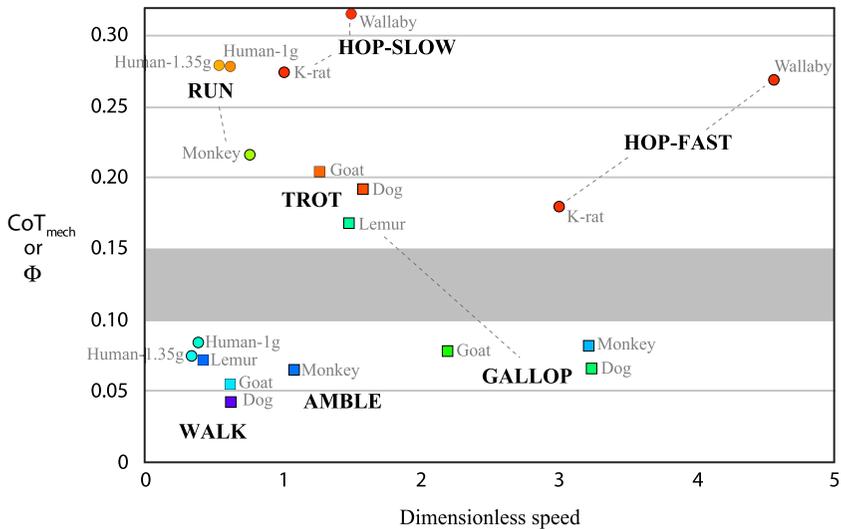


FIGURE 2.1.3 Mechanical cost of transport as a function of dimensionless speed in bipedal and quadrupedal mammals (Lee et al., 2013).

that of walking (Fig. 2.1.3; Lee et al., 2013). The lower mechanical cost of walking is achieved by the combined action of trailing and leading leg forces, which exert propulsive and then braking forces during the downward to upward redirection of the center mass.

As predicted from the theoretical observation that mechanical cost of transport is inversely proportional to the number of collisions (Ruina et al., 2005), quadrupeds more nearly approach zero-collision locomotion than bipeds. Collision-based analysis shows that walking dogs achieve a mechanical cost of transport approximately half that of humans (Fig. 2.1.3). Faster quadrupedal gaits such as the gallop and amble also use sequenced leg contacts and favorable timing of braking and propulsive forces to reduce the work of redirecting the center of mass—but not to the same extent as in quadrupedal walking (Fig. 2.1.3). One might ask if terrestrial animals with six, eight, or ten legs can achieve even lower mechanical costs of transport than quadrupeds, however, most studies of multilegged locomotion in cockroaches and crabs report SLIP-like bouncing gaits akin to quadrupedal trotting (reviewed by Holmes et al., 2006). If multilegged animals combined sequenced leg contacts with propulsive then braking forces during the downward to upward transition, it is plausible that animals with many legs could more smoothly redirect the center of mass, thereby achieving a lower mechanical cost of transport than bipeds or quadrupeds.

Brachiation of gibbons (Usherwood and Bertram, 2003) and siamangs (Michilsons et al., 2012) provide another example of collision-based dynamics that reduce mechanical cost. Using a single arm to redirect the center of mass from downward to upward, these apes achieve mechanical costs of transport lower than those of walking quadrupeds. Collision-based mechanical cost is mitigated more readily during suspensory locomotion because the arm can pull on the overhead support to exert propulsive then braking force during downward to upward redirection of the center of mass. As shown in bipedal, quadrupedal, and brachiation examples, this pattern of force is the hallmark of collision-based dynamics, which seek to redirect the center of mass using the least mechanical work—in accordance with D’Alembert’s principle of orthogonal constraint.

2.1.2 EFFECTS OF SIZE

Size is a fundamental determinant of structure and function in animal and machines alike. For the scaling of legged locomotion, the principle of dynamic similarity is a key concept that was introduced four decades ago as a model to predict gait characteristics of bipedal dinosaurs based upon locomotor data from extant birds and humans (Alexander, 1976). The same construct was later applied to quadrupedal mammals (Alexander and Jayes, 1983). The dynamic similarity hypothesis holds that animals of different sizes moving at the same dimensionless speed tend to use the same dimensionless stride lengths, stride frequencies, duty factors and maximum forces. Forward speed is normalized as the Froude number, or preferably the square root of Froude number (McMahon and Cheng, 1990), known as dimensionless speed U :

$$U = \bar{v}_y / \sqrt{gh}, \quad (2.1.2)$$

where \bar{v}_y is the mean forward velocity, g is gravitational acceleration, and h is hip height or leg length. The Froude number represents the ratio of inertial to gravitational acceleration, indicating that a rigid inverted pendulum would escape its circular trajectory at a Froude number or dimensionless speed greater than one. Walking bipeds and quadrupeds, however, abandon walking gaits at Froude numbers greater than about 0.5, corresponding to dimensionless speeds greater than about 0.7 (Alexander, 1984). Dimensionless speeds are determined in the same way for running, hopping, trotting, and galloping gaits and, because these gaits often exceed a dimensionless speed of one, using Froude number instead of its square root would show substantial nonlinearity with increasing speed.

A refinement of Alexander’s rigid inverted pendulum calculation predicts a different boundary for escape from a circular arc for a given combination of

speed and step length, here defined as the distance between right and left foot-falls. For example, a maximum dimensionless step length (i.e., step length relative to leg length) of 1.15 can be achieved by a rigid-legged walker at a dimensionless speed of 0.7, approximating the fastest walking of humans (Usherwood, 2005). Adding an impulsive step-to-step transitions and a minimum mechanical cost criterion, a subsequent computer optimization study showed that rigid-legged walking is optimal only at dimensionless step lengths less than 0.76 (Srinivasan and Ruina, 2006). Despite observations that humans use intermediate dimensionless step lengths of about 1.0 at a dimensionless speed of 0.7, there is no evidence that human walking follows the circular arc of a rigid inverted pendulum. In fact, experimental data show that vertical oscillations of the center of mass during fast human walking are just 17–28% of those predicted by rigid-legged walking models (Lee and Farley, 1998). In light of these observations, a different mechanism might be found to explain the relationship of dimensionless step length to maximum walking speed of humans.

Dynamic similarity is used to determine equivalent speeds in animals of different leg length such that other effects on locomotion, such as leg number, gait, and phylogeny can be better understood. The principle of dynamic similarity normalizes stride length to hip height h , stride frequency to $\sqrt{g/h}$, and force to body weight mg , predicting equal values of these dimensionless parameters at a given dimensionless speed. Duty factors, calculated as the ratio of foot contact period to stride period, are also predicted to be equal at the same dimensionless speed. When bipedal and quadrupedal gaits are normalized according to dynamic similarity, these dimensionless parameters tend to follow similar, yet sometimes offset trend-lines across a range of dimensionless speeds from 0.2 to 4.0 (Alexander, 1976, 1984, 2004; Alexander and Jayes, 1983). For example, quadrupedal primates tend to use longer stride lengths than other quadrupeds at a given dimensionless speed. Likewise, at the same dimensionless speed, humans walking and running bipedally use much shorter dimensionless stride lengths than do chimpanzees and bonobos during quadrupedal gaits (Aerts et al., 2000). A comparative study of bipedal locomotion in birds spanning three orders of magnitude in size applied dynamic similarity to show that small birds tend to use relatively longer stride lengths and lower frequencies than large birds at a given dimensionless speed (Gatesy and Biewener, 1991). Dynamic similarity is useful for determining equivalent speeds amongst subjects of different size, such as in a human gait study, and it is also well suited to the analysis of species that span a substantial size range. It should be used as an initial hypothesis for understanding size effects in legged locomotion rather than a precise predictive model, for example, across different species or in hopping versus striding locomotion.

Dynamic similarity has also been applied to reduced- and hyper-gravity studies of legged locomotion, wherein gravity, as well as hip height, are free variables influencing dimensionless speed and stride frequency. A study of simulated reduced gravity during human walking, found that dimensionless stride length at a given dimensionless speed decreased as gravity was reduced from 1.0 g to 0.25 g, violating the dynamic similarity prediction that stride length should remain the same at equal dimensionless speeds (Donelan and Kram, 1997). However, in agreement with dynamic similarity, the same study showed that duty factor at a given dimensionless speed was unchanged across gravity conditions. A study of simulated hypergravity during human walking showed qualitative agreement with dynamic similarity, where duty factor increased and dimensionless stride length decreased as dimensionless speed was decreased by a 1.35 g hypergravity condition (Lee et al., 2013). Reduced- and hypergravity studies of human running also show trends consistent with dynamic similarity (Donelan and Kram, 1997; Minetti, 2001). Hence, the principle of dynamic similarity seems robust to changes in gravity with the exception that stride lengths are unexpectedly shortened by reduced gravity conditions during walking. Dynamic similarity therefore remains the primary model to determine comparable speeds whenever animal size or gravity conditions are variable.

Another method of accounting for speed effects that has been used in studies of quadrupedal mammals is to target the trot–gallop transition as a “physiologically equivalent” speed for animals of different size. This approach was introduced in a study showing that stride frequency scales as body mass to the -0.14 power and stride length, to the 0.38 power in mammals from mice to horses (Heglund et al., 1974). A comparable approach measured these parameters at the fastest experimental speeds of running birds and humans to show that stride frequency scales as body mass to the -0.18 power and stride length to the 0.38 power (Gatesy and Biewener, 1991). Trot–gallop transition speeds have also been used to compare oxygen consumption rate and mechanical power (Heglund et al., 1982; Taylor et al., 1982), as well as in vivo bone strain and effective mechanical advantage of muscles about joints (Biewener, 1989, 1990) across quadrupeds of vastly different size. As predicted by dynamic similarity, mechanical cost of transport at the trot–gallop transition speed is invariant across quadrupedal mammals from mice to horses, and also at corresponding speeds of bipedal and multilegged runners (Full, 1989; Full and Tu, 1991). The mechanical cost of transport determined by this allometric analysis is 0.1 based upon positive work alone—doubling this value to account for negative work equals CoT_{mech} as defined in Eq. (2.1.1). Running, hopping, and trotting usually show a CoT_{mech} near 0.2. Yet, as already discussed, mechanical cost of transport is several-fold lower for gaits that use collision-based mitigation of work, such as walking, ambling, and galloping (Fig. 2.1.3).

Animal size also influences the stiffness of the modeled stance leg (or legs) during bipedal running, hopping, and trotting gaits. The spring-loaded inverted pendulum (SLIP) is the simplest two-dimensional model of leg compliance, as it imagines such a virtual spring-loaded leg acting between the center of mass and the ground (McMahon and Cheng, 1990). Because this method depends only on measurement of the total force vector during leg contact and a kinematic estimate of initial virtual leg length, it can be applied to quadrupedal and hexapedal trotting as well as to bipedal running and hopping (Farley et al., 1993; Blickhan and Full, 1993). Considering eight mammals spanning three orders of magnitude in body mass and including hoppers, trotters, and a human runner, allometric analysis showed that virtual leg stiffness scales as body mass to the two-thirds power. This relationship matches the expected leg stiffness based on the ratio of force, which scales in direct proportion to body mass, to length, which scales as body mass to the one-third power when geometric similarity is assumed.

A more explicit experimental approach measures leg stiffness by tracking position of the proximal joint (the hip or shoulder) and modeling a radial leg that extends through the distal-most joint to the ground. This method measures radial leg stiffness by placing an actuator in series with a modeled leg spring and choosing the spring constant that minimizes actuator work. Considering five mammalian species, radial leg stiffness scales approximately as body mass to the two-thirds but is more than 30% stiffer than the virtual leg spring at a given body mass (Lee et al., 2014). The scaling of leg spring constants has also been analyzed using a minimum work criterion in an actuated, damped SLIP model (Birn-Jeffery et al., 2014). This model successfully reproduced the running dynamics of five striding bird species ranging in size from quail to ostriches and found a dimensionless leg stiffness invariant with body mass, as did the analysis of Blickhan and Full (1993). All modeling approaches used so far to investigate the scaling of leg spring stiffness show that stiffness is a function of body mass to the two-thirds power, or equivalently, that dimensionless stiffness is invariant with size.

2.1.3 SUMMARY

This section has examined stance as a locomotor subfunction where a leg or legs redirect the center of mass through simultaneous and/or sequenced contacts with the substrate. The cost of this redirection is determined by collision-based dynamics. Whether there is an aerial phase or not, the summed vertical ground reaction force rises above body weight during part of the stride to redirect the center of mass from downward to upward. This is achieved by a single contact leg in bipedal running and by simultaneous contact of more than one leg

in all other gaits. During running, hopping, and trotting, downward to upward redirection of the center of mass is SLIP-like, with braking followed by propulsion. This pattern is reversed in walking, ambling, and galloping gaits, with propulsion followed by braking achieved by sequenced contacts of more than one leg. Mechanical work can be measured using a collision-based approach, which considers the relationship between the center of mass velocity vector and the overall force vector. Whenever these vectors are perpendicular, no work is done on the center of mass. Mechanical work is minimized in this way by walking, ambling, and galloping but not in SLIP-like gaits. Hence, the mechanical cost of transport is about three-fold greater during SLIP-like running, hopping, and trotting compared with gaits that mitigate work. Theory and some experimental evidence suggest that work is increasingly mitigated as the number of sequenced leg contacts increases. The principle of dynamic similarity estimates equivalent speeds for animals of vastly different size by determining a dimensionless speed according to the square root of leg length. This model considers dimensionless stride length, frequency, force, and mechanical cost of transport—predicting equal values of these parameters at a given dimensionless speed. When size spans orders of magnitude, dimensional parameters can also be expressed as power-functions of body mass and this approach has yielded scaling relationships for stride length, frequency, force magnitude, spring stiffness, and mechanical cost of transport. Stance also influences and is influenced by the swing and balance locomotor subfunctions discussed in the remaining sections of this chapter.

REFERENCES

- Aerts, P., Van Damme, R., Van Elsacker, L., Duchêne, V., 2000. Spatio-temporal gait characteristics of the hind-limb cycles during voluntary bipedal and quadrupedal walking in bonobos (*Pan paniscus*). *Am. J. Phys. Anthropol.* 111 (4), 503.
- Alexander, R., 1976. Estimates of speeds of dinosaurs. *Nature* 261, 129–130.
- Alexander, R.M., 1984. The gaits of bipedal and quadrupedal animals. *Int. J. Robot. Res.* 3 (2), 49–59.
- Alexander, R., 2004. Bipedal animals, and their differences from humans. *J. Anat.* 204 (5), 321–330.
- Alexander, R., Jayes, A.S., 1983. A dynamic similarity hypothesis for the gaits of quadrupedal mammals. *J. Zool.* 201 (1), 135–152.
- Bertram, J.E.A., Gutmann, A., 2009. Motions of the running horse and cheetah revisited: fundamental mechanics of the transverse and rotary gallop. *J. R. Soc. Interface* 6 (35), 549–559.
- Biewener, A.A., 1989. Scaling body support in mammals: limb posture and muscle mechanics. *Science* 245 (4913), 45–48.
- Biewener, A.A., 1990. Biomechanics of mammalian terrestrial locomotion. *Science* 250 (4984), 1097.
- Biewener, A.A., 2005. Biomechanical consequences of scaling. *J. Exp. Biol.* 208 (9), 1665–1676.
- Birn-Jeffery, A.V., Hubicki, C.M., Blum, Y., Renjewski, D., Hurst, J.W., Daley, M.A., 2014. Don't break a leg: running birds from quail to ostrich prioritise leg safety and economy on uneven terrain. *J. Exp. Biol.* 217 (21), 3786–3796.

- Blickhan, R., 1989. The spring–mass model for running and hopping. *J. Biomech.* 22 (11–12), 1217–1227.
- Blickhan, R., Full, R.J., 1993. Similarity in multilegged locomotion: bouncing like a monopode. *J. Comp. Physiol. A* 173 (5), 509–517.
- Cavagna, G.A., Thys, H., Zamboni, A., 1976. The sources of external work in level walking and running. *J. Physiol.* 262 (3), 639.
- Cavagna, G.A., Heglund, N.C., Taylor, C.R., 1977. Mechanical work in terrestrial locomotion: two basic mechanisms for minimizing energy expenditure. *Am. J. Physiol., Regul. Integr. Comp. Physiol.* 233 (5), R243–R261.
- Daeschler, E.B., Shubin, N.H., Jenkins, F.A., 2006. A Devonian tetrapod-like fish and the evolution of the tetrapod body plan. *Nature* 440 (7085), 757–763.
- D’Alembert, J.L.R., 1743. *Traité de dynamique*.
- Dickinson, M.H., Farley, C.T., Full, R.J., Koehl, M.A.R., Kram, R., Lehman, S., 2000. How animals move: an integrative view. *Science* 288 (5463), 100–106.
- Donelan, J.M., Kram, R., 1997. The effect of reduced gravity on the kinematics of human walking: a test of the dynamic similarity hypothesis for locomotion. *J. Exp. Biol.* 200, 3193–3201.
- Farley, C.T., Glasheen, J., McMahon, T.A., 1993. Running springs: speed and animal size. *J. Exp. Biol.* 185 (1), 71–86.
- Full, R.J., 1989. Mechanics and energetics of terrestrial locomotion: bipeds to polypeds. In: *Energy Transformations in Cells and Animals*, pp. 175–182.
- Full, R.J., Tu, M.S., 1991. Mechanics of a rapid running insect: two-, four- and six-legged locomotion. *J. Exp. Biol.* 156 (1), 215–231.
- Gatesy, S.M., Biewener, A.A., 1991. Bipedal locomotion: effects of speed, size and limb posture in birds and humans. *J. Zool.* 224 (1), 127–147.
- Genin, J.J., Willems, P.A., Cavagna, G.A., Lair, R., Heglund, N.C., 2010. Biomechanics of locomotion in Asian elephants. *J. Exp. Biol.* 213 (5), 694–706.
- Geyer, H., Seyfarth, A., Blickhan, R., 2006. Compliant leg behaviour explains basic dynamics of walking and running. *Proc. R. Soc. Lond. B, Biol. Sci.* 273 (1603), 2861–2867.
- Gray, J., 1968. *Animal Locomotion*. Weidenfeld & Nicolson.
- Griffin, T.M., Main, R.P., Farley, C.T., 2004. Biomechanics of quadrupedal walking: how do four-legged animals achieve inverted pendulum-like movements? *J. Exp. Biol.* 207 (20), 3545–3558.
- Heglund, N.C., Taylor, C.R., McMahon, T.A., 1974. Scaling stride frequency and gait to animal size: mice to horses. *Science* 186 (4169), 1112–1113.
- Heglund, N.C., Fedak, M.A., Taylor, C.R., Cavagna, G.A., 1982. Energetics and mechanics of terrestrial locomotion. IV. Total mechanical energy changes as a function of speed and body size in birds and mammals. *J. Exp. Biol.* 97 (1), 57–66.
- Hildebrand, M., 1965. Symmetrical gaits of horses. *Science* 150 (3697), 701–708.
- Holmes, P., Full, R.J., Koditschek, D., Guckenheimer, J., 2006. The dynamics of legged locomotion: models, analyses, and challenges. *SIAM Rev.* 48 (2), 207–304.
- Kuo, A.D., 2002. Energetics of actively powered locomotion using the simplest walking model. *J. Biomech. Eng.* 124 (1), 113–120.
- Lee, C.R., Farley, C.T., 1998. Determinants of the center of mass trajectory in human walking and running. *J. Exp. Biol.* 201 (21), 2935–2944.
- Lee, D.V., Bertram, J.E., Todhunter, R.J., 1999. Acceleration and balance in trotting dogs. *J. Exp. Biol.* 202 (24), 3565–3573.
- Lee, D.V., Bertram, J.E., Anttonen, J.T., Ros, I.G., Harris, S.L., Biewener, A.A., 2011. A collisional perspective on quadrupedal gait dynamics. *J. R. Soc. Interface*, rsif20110019.
- Lee, D.V., Comanescu, T.N., Butcher, M.T., Bertram, J.E., 2013. A comparative collision-based analysis of human gait. *Proc. R. Soc. Lond. B, Biol. Sci.* 280 (1771), 20131779.
- Lee, D.V., Isaacs, M.R., Higgins, T.E., Biewener, A.A., McGowan, C.P., 2014. Scaling of the spring in the leg during bouncing gaits of mammals. *Integr. Comp. Biol.* 54 (6), 1099–1108.

- McMahon, T.A., Cheng, G.C., 1990. The mechanics of running: how does stiffness couple with speed? *J. Biomech.* 23, 65–78.
- Michilzens, F., D'Août, K., Vereecke, E.E., Aerts, P., 2012. One step beyond: different step-to-step transitions exist during continuous contact brachiation in siamangs. *Biol. Open* 1 (5), 411–421.
- Minetti, A.E., 2001. Invariant aspects of human locomotion in different gravitational environments. *Acta Astronaut.* 49 (3), 191–198.
- Murphy, K.N., Raibert, M.H., 1985. Trotting and bounding in a planar two-legged model. In: *Theory and Practice of Robots and Manipulators*. Springer US, pp. 411–420.
- Panganiban, G., Irvine, S.M., Lowe, C., Roehl, H., Corley, L.S., Sherbon, B., Wray, G.A., et al., 1997. The origin and evolution of animal appendages. *Proc. Natl. Acad. Sci.* 94 (10), 5162–5166.
- Ruina, A., Bertram, J.E., Srinivasan, M., 2005. A collisional model of the energetic cost of support work qualitatively explains leg sequencing in walking and galloping, pseudo-elastic leg behavior in running and the walk-to-run transition. *J. Theor. Biol.* 237 (2), 170–192.
- Shubin, N.H., Daeschler, E.B., Jenkins, F.A., 2014. Pelvic girdle and fin of *Tiktaalik roseae*. *Proc. Natl. Acad. Sci.* 111 (3), 893–899.
- Srinivasan, M., Ruina, A., 2006. Computer optimization of a minimal biped model discovers walking and running. *Nature* 439 (7072), 72–75.
- Taylor, C.R., Heglund, N.C., Maloiy, G.M., 1982. Energetics and mechanics of terrestrial locomotion. I. Metabolic energy consumption as a function of speed and body size in birds and mammals. *J. Exp. Biol.* 97 (1), 1–21.
- Usherwood, J.R., 2005. Why not walk faster? *Biol. Lett.* 1 (3), 338–341.
- Usherwood, J.R., Bertram, J.E., 2003. Understanding brachiation: insight from a collisional perspective. *J. Exp. Biol.* 206 (10), 1631–1642.

Chapter 2.2

Leg Swinging

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For successful locomotion, the swing leg needs to be prepared for the next landing event. For a specific or changing gait condition (gait type, speed, environment), swing leg movement could require achieving a certain foot placement (e.g., to avoid hitting a pothole, to recover from a large perturbation). However, in most cases the actual location of the foot during contact is not a specific target of control. Then, the leg swinging as one locomotion subfunction can rely on the system dynamics to result in a steady gait pattern.

Based on these two mechanisms (foot placement and exploration of natural swing leg dynamics) leg swinging can fulfill different scenarios. In the following we will describe how leg swinging contributes to locomotion, namely how it interacts with other locomotor subfunctions as well as its role in perturbation recovery and switching between the gaits.

At the level of locomotion control, these two mechanisms can be compared to position control vs. passive dynamic walking (Kuo, 2007a). In legged robots, ZMP (zero moment point; Vukobratovic and Borovac, 2004) is one of the most common approaches to achieve stable locomotion in systems employing positional control of leg joints (e.g., in Asimo; Sakagami et al., 2002). ZMP refers to the point inside the base of support about which the ground contact forces exert no moment (see Subchapter 2.3). The idea behind *dynamic walking* relies on passive dynamics of the legs to produce walking, avoids position control, and focuses on producing a cyclic gait (Miura and Shimoyama, 1984; McGeer, 1990a, 1990b; Collins et al., 2005). In passive dynamic walking robots, the stance leg and swing leg behave as an inverted and regular pendulum, respectively (for more explanation about passive dynamic walking see Subchapter 4.6). The required energy to compensate the losses is generated either by gravity when the robot walks on gently sloped terrain (McGeer, 1990a) or minimal actuation (Collins et al., 2005). In Kuo (2007a) the term dynamic walking is defined to refer “specifically to machines designed to harness leg dynamics, using control more to shape and tune these dynamics than to impose prescribed kinematic motions.” Such an actuation can be provided by hip torque (Collins et al., 2005) or push off with the trailing leg’s ankle (Kuo, 2002). Therefore, the relation between leg swinging with stance control and/or posture control will come to account.

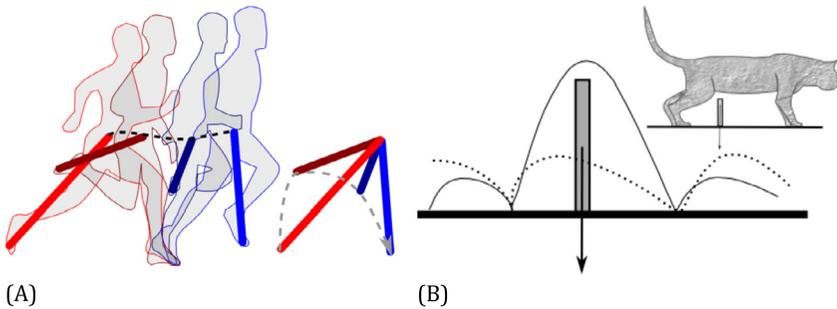


FIGURE 2.2.1 (A) Swing leg movement in human running with changing leg length and leg angle. (B) Leg swinging in cat walking (McVea and Pearson, 2006). Cats stepped over an obstacle with the forelegs. After a 20 s delay, hind foot toe trajectory showed that the animal remembered the obstacle (solid line), which was lowered while the animal stood still. The dotted line shows a step without the obstacle.

2.2.1 CHARACTERIZING FEATURES OF LEG SWINGING

Leg swinging can be defined as rotational swing leg motion with complementary axial movement (Fig. 2.2.1A), e.g., for ground clearance in walking or for reducing swing leg moment of inertia in running. Indeed, this axial leg shortening is very important in special situations like hurdle running. In that sense, leg swinging can be considered as control of the end effector (here the foot) of a manipulator. In cats, this foot trajectory to overcome obstacles was found to be memorized. The pattern could be restored and realized with the hind limbs after locomotion was interrupted for substantial time as shown in Fig. 2.2.1B (McVea and Pearson, 2006).

However, the main role of leg swinging in (unperturbed) locomotion is its rotational movement, which results in a reorientation of the leg in preparation for the next contact phase. This primitive function of the (load-released) swing leg can already be found in newborns; it is known as the stepping reflex (Siegler et al., 2006).

Swing leg movements and its related control can be characterized by the leg's states at touchdown (landing condition) and the foot trajectory during swing phase (ground clearance of the foot). The leg orientation and leg length at touch-down also contributes to the dynamics of the next contact phase and thus to other locomotion subfunctions (stance and balance). Not only the leg configuration at touchdown but also its changes with time (angular velocity and axial speed of leg shortening/extension) is an important feature to describe gait dynamics. Thus, leg swinging at touchdown is characterized by leg length, leg angle with respect to ground (angle of attack) (Seyfarth et al., 2001), leg angular speed (Seyfarth et al., 2003) and leg shortening speed (Blum et al., 2010). In both walking and running, the swing-leg moves backward towards

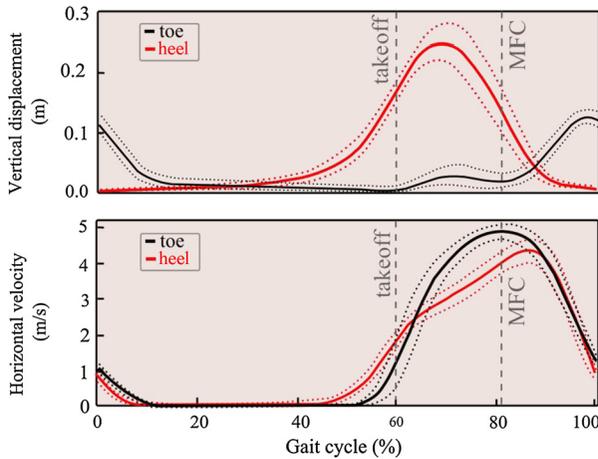


FIGURE 2.2.2 Vertical displacement and horizontal velocity of heel and toe in walking. Minimum foot clearance (MFC) coincides with the moment that the foot travels with maximum horizontal velocity. The figure is adopted from Winter (1992).

the ground before touchdown (late swing-phase) (Muybridge, 1955), called swing leg retraction (SLR). This backward movement is represented by positive angular velocities of the leg (Poggensee et al., 2014) and has a large contribution to gait stability (Seyfarth et al., 2003). SLR supports ground speed matching, helps reduce impact losses during landing (De Wit et al., 2000; Blum et al., 2010) and maintain forward locomotion speed.

The foot trajectory during swing phase needs to provide sufficient ground clearance. In human walking, the foot of the swing leg is aligned horizontally with only small distance (1–2 cm) to the ground which reduces with age increasing (Winter, 1992). In contrast to Winter claim, Mills et al. showed that increasing the variability in minimum toe clearance (MTC) results in high risk of tripping in the elderly while the MTC medians in two young and elderly men were similar (Mills et al., 2008). This helps generate stable and energy efficient gait patterns (Wu and Kuo, 2015). Minimum foot clearance (MFC) is a critical event because the foot gets its maximum horizontal velocity and lowest height simultaneously (shown in Fig. 2.2.2) which increases the danger of tripping in case of hitting an obstacle (Begg et al., 2007).

Judging from human leg muscle activities in the swing leg movement, biarticular hip muscles rectus femoris (RF) and hamstrings (HA) seem to be the main contributors in the swing phase of walking (Nilsson et al., 1985). By modeling these two muscles with biarticular springs, we aim at a better mechanical understanding of their activities in producing stable gait. In addition, such a passive mechanism may also replicate strong correlation observed between RF and

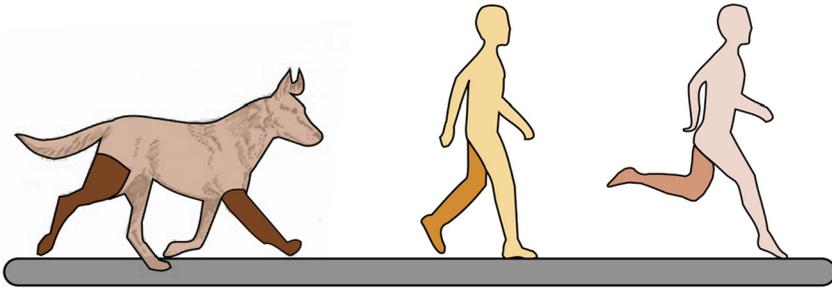


FIGURE 2.2.3 Leg swinging in different gaits. Dark colors show the swing legs.

HA in human swing leg movement (Prilutsky et al., 1998), as a consequence of body mechanics. In Sect. 2.2.3 we introduce a model for swing leg control based on this observation.

2.2.2 LEG SWINGING EFFECTS IN LOCOMOTION

Leg swinging is required for robust locomotion in order to initiate the next step (Fig. 2.2.3). This is a consequence of the limited number of legs which need to hit the ground in a sequential manner. Leg swinging contributes to locomotion dynamics in many ways:

- (i) Determining stance phase dynamics as a result of the landing condition,
- (ii) Shaping the system states to achieve versatile gaits with selected gait type, footfall pattern, step length, step frequency, robustness and efficiency,
- (iii) Distribution of energies in forward, lateral, and vertical directions, e.g., acceleration or changes in locomotion direction,
- (iv) Overcoming unwanted ground contacts and perturbation recovery, e.g., obstacle avoidance.

2.2.2.1 Contributing to Stance Phase Dynamics

When determining the initial condition of the next stance leg, swing leg movement significantly affects the stance phase dynamics. It is related not only to the leg configuration, but also to the momentum and angular speed which initiate the new states after impact. Human and animal locomotion experiments show high sensitivity of the initial leg loading during stance to its landing conditions (Moritz and Farley, 2004; Birn-Jeffery and Daley, 2012; Daley and Biewener, 2011). Daley and Biewener (2006) showed that the variation in leg contact angles explains 80% of the variation in stance impulse after an unexpected pothole in guinea fowl running.

2.2.2.2 Trade-off Between Versatility, Robustness, and Efficiency

Redirection of the center of mass speed at touchdown can be used by a suitable swing leg control (foot placement) to stabilize the gait (Townsend, 1985). Furthermore, Winter et al. claim that foot placement is a precise and multi-factorial motor control task which is required (beside stance leg control) for stable gaits (Winter, 1992). Donelan et al. showed that both average external mechanical work and metabolic rates increase with the fourth power of step length in human walking (Donelan et al., 2002), which confirms importance of foot placement in energy optimization. Other investigations in humans and quadrupeds show that energy consumption is optimized in many legged animals' locomotion (Alexander, 1984). In the steady state condition, biological locomotors consume the least energy for leg swinging and benefit from passive dynamics. For example, the positive work of human muscles is relatively small (compared to stance) (Neptune et al., 2008), and in hip muscles, they partly function as actively-tunable springs (Doke et al., 2005). However, if perturbation happens or when humans walk on rough terrains, the energy consumption increases considerably to achieve stability (Voloshina et al., 2013). Therefore, one significant contribution of the swing leg adjustment is balancing a trade-off between versatility, robustness, and energy consumption.

As mentioned before, leg retraction is an important feature of leg swinging which has a significant effect on movement stability in quadrupeds (Herr and McMahon, 2001), birds (Daley et al., 2007), bipedal biological locomotors (Muybridge, 1955; Gray, 1968; Blum et al., 2010), and robots (Wisse et al., 2005). This technique for swing leg control reduces foot-velocity with respect to the ground and, as a result decreases landing impact (De Wit et al., 2000). Increasing the leg length in late swing and also angular accelerations (Vejdani et al., 2013) can improve stability and robustness (Blum et al., 2010). In human walking and running, there is a linear relationship between motion speed and swing leg retraction speed and acceleration (Fig. 2.2.4, Poggensee et al., 2014).

2.2.2.3 Distribution of Energies in Forward, Lateral, and Vertical Directions

Adjusting the leg parameters during flight/swing phase is more energy efficient than stance phase because end of the leg (the foot) is free to move. Since the swing leg movement initiates the states of the stance leg in the next step, tuning the system states to select the limit cycle is performed easier during leg swinging. The resulting redirection of the energy in different directions can be used in changing (a) the forward speed, (b) the gait, (c) foot placement on a specific target/position on the ground (e.g., walking on large stones), (d) motion direction, (e) lateral balance, and (f) locomotion on slopes and stairs.

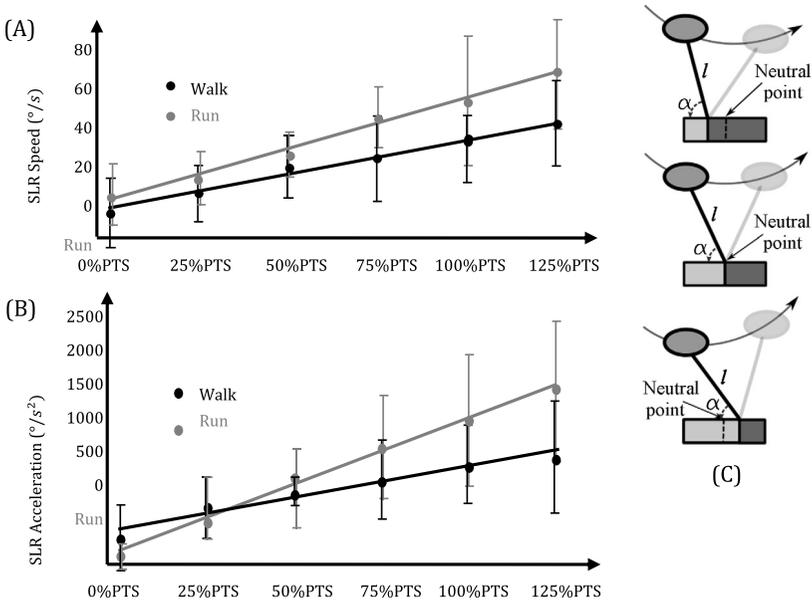


FIGURE 2.2.4 Swing leg retraction (A) speeds and (B) accelerations for human walking and running. Data points are the global means at each speed with errors bars of one standard deviation and associated trend lines for 21 subjects. Figures are adopted from [Poggensee et al. \(2014\)](#). (C) Swing leg adjustment effect on gait speed using neutral point concept. Foot placing at neutral point results in symmetric gait and keeping motion speed. Magnitude and direction of displacement of foot and the neutral point determine the magnitude and direction of forward acceleration. Dark and light boxes show acceleration and deceleration phase, respectively.

(a) Changing the speed (velocity control)

Studies on human walking and running show that locomotion speed significantly influences leg angle of attack and leg retraction speed as two features of swing leg adjustment ([Sharbafi et al., 2013, 2016, 2017](#)). A potential role of leg adjustment for control of forward speed is indicated by modeling and robotic studies ([Dunn and Howe, 1996](#)). This equally holds for different types of conceptual gait models (explained in detail in Chap. 3 and 4), e.g., inverted pendulum, spring loaded inverted pendulum, or rimless wheels, where angle of attack is a key control property in speed adjustment.

The concept of *neutral point* introduced by [Raibert \(1986\)](#) shows such a relation between swing leg adjustment and motion speed in a simple model of running (describing a monopod or biped hopping robot). Neutral point is the foot position (relative to the hip) at touchdown which results in symmetric leg movement around mid-stance. Then the forward speed is the same at liftoff as it is at touchdown and the average horizontal and angular acceleration over each

stride becomes zero. With deviation from the *neutral point*, the body accelerates, with the magnitude and direction of acceleration proportional to the magnitude and direction of the displacement between foot position and neutral point, as shown in Fig. 2.2.4C. Based on that, he suggested the following controller to adjust forward running speed in monopod hopper and biped runner:

$$\alpha = \arccos\left(\frac{\dot{x}T_s + 2k(\dot{x} - \dot{x}_d)}{2l}\right), \quad (2.2.1)$$

in which α , T_s , l , k , \dot{x} , and \dot{x}_d are angle of attack, stance time, leg length, constant gain, current and desired speed, respectively. Although this controller has limitations at high speed running, it can perfectly demonstrate the ability of swing leg control in gait speed adjustment in practice.

(b) Changing the gait

Raibert showed that with a simple leg adjustment protocol (beside axial leg force control) a single-legged (monopod) robot can stably hop, run, and even summersault (Raibert, 1986). For switching from one type of locomotion to another (e.g., walking to running) or from standing to dynamic locomotion (and vice versa) leg swinging plays an important role. Two different cases of gait switching are between:

1. Standing and Walking/Running

Gait initiation and termination are two types of switching between standing and locomotion. In gait initiation, the task is changing from keeping balance (placing the body CoM within base of support) to moving body over the ground. During this transition, the upcoming swing leg prepares its push-off by actively loading and then unloading the limb. Consider the inverted pendulum model to represent the human body to explain this phenomenon. Assume the right and left legs are the upcoming swing and stance legs in the first step, respectively. In order to release the right leg from ground contact, an angular momentum is required to shift the center of mass to above the other leg (Winter, 1995). As a result, an increase in GRF is observed at the right leg to generate the required angular momentum and the center of pressure is moved first from middle of the two feet to the right and then to the left foot (Winter, 1995). Compared to gait initiation, COM trajectories are mirrored during gait termination (Jian et al., 1993). This also holds for COP trajectories. *Capture point* and *capture region* concepts (Pratt and Tedrake, 2006) describe how models help calculate a region to place the swing leg to stop in one (or more) step(s) (see Section 2.2.3 for details).

2. Walking and Running

For a given speed (e.g., when moving on a treadmill), either walking or running is preferred (Thorstensson and Robertson, 1987; Hreljac, 1993). If the

average speed (i.e., the traveling time) over a specific distance matters, humans sometimes prefer a mixture of walking and running (Long and Srinivasan, 2013). Long and Srinivasan showed that for moving between two fixed points in a specific time, walking and running are preferred for long and short travel time, respectively, but a mixture of walking and running and even standing is chosen for intermediate time. Thus, sometimes steady locomotion may not be energy optimal. This finding was also shown analytically with computational optimization (Long and Srinivasan, 2013). However, humans prefer to switch between walking and running at a specific speed called PTS (preferred transition speed) which barely changes at different positive or negative accelerations (Segers, 2006). Walking is more efficient at speeds below PTS, while running is the more optimal gait for moving faster than PTS. In running at speeds above PTS, muscle activations in the swing leg are lower than in walking at the same speed (Prilutsky and Gregor, 2001). In contrast, in walking at speeds below PTS the muscle activations in stance leg are less than in running at the same speed (Prilutsky and Gregor, 2001). Therefore, costs of swing leg and stance leg movements might be the critical term in determining the energy consumption at high and low speeds, respectively. The transition between walking and running can be related to muscle functions as at maximum walking speed both hip and ankle muscles reach their limits in force production (Neptune and Sasaki, 2005; Prilutsky and Gregor, 2001). These limits are resolved by switching from walking to running at the same speed. Hence, changing gaits support efficient muscle function at different speeds similar to the function of the gear in a bicycle.

(c) Targeting

In steady state gaits with no targets for foot placement (nontargeted gait) swing leg control usually determines the angle of attack (Seyfarth et al., 2001) and leg retraction (Seyfarth et al., 2003; Herr et al., 2002) in a periodic manner. To reach certain targets, swing leg control can be used for foot placement and adjusting the foot orientation. This results in different kinds of foot contacts (e.g., heel strike or fore foot) at touchdown and may help overcome unwanted ground contacts (e.g., with obstacles). Passing over a river by placing the feet on stones or hurdle running are extreme cases of targeting, needing precise swing leg control which do not regularly happen in daily activities. More frequent applications of swing leg adjustment for targeting can be found in locomotion on rough terrains, stepping over obstacles or turning the motion direction. Using fore-foot touchdown for impact avoidance or walking on rough terrains are samples of swing leg strategies for targeting (Lieberman et al., 2010; Voloshina et al., 2013; Pratt and Drake, 2006).

(d) Steering

Steering the motion occurs in 3D space. Swing leg control in lateral plane plays the main role in steering the gait besides upper body yaw movement using stance leg. [Maus and Seyfarth \(2014\)](#) showed how lateral leg adjustment can compensate leg asymmetry and develop walking in circles or on a straight line.

(e) Lateral balance with foot placement (walking)

Lateral balancing is much more challenging than balancing in the sagittal plane ([Bauby, 2000](#)). McGeer demonstrated that passive walking dynamics allow descending a gentle slope without external power ([McGeer, 1990a](#)). So in sagittal plane, the passive leg adjustment and interaction between dynamics of two legs establish a periodic gait down a slight incline, with no external input except gravity. However, this does not hold in frontal plane. Here, a lateral control scheme is needed to withstand perturbations ([Kuo, 1999](#)). [Bauby and Kuo \(2000\)](#) claim that unlike for fore-aft stability high-level neural feedback control is necessary for maintaining lateral stability. The support of body weight requires stabilizing motion dynamics in sagittal plane ([MacKinnon and Winter, 1993](#)). However, to achieve stability in frontal plane, proper sensing of lateral motion (like visual and vestibular input) is required for perturbation recovery ([Warren et al., 1996](#); [Winter, 1995](#)).

(f) Handling gravity effect (uphill–downhill, stair climbing)

Locomotion on inclined ground or on stairs requires different swing leg adjustment strategies (different trajectories) compared to level ground walking. Foot placement in sloped terrains is more complex than on flat ground as both step length and step height need to be controlled. For example, trajectories generated using a simple pendulum like model of swing leg are not feasible in (stair) ascending because in this case vertical lifting is also required besides horizontal forward movement. In that respect, additional parameters describing the environment (ground) are required. This increases complexity of swing leg control which contributes to both ground clearance and energy management. The control strategies to cope with ground level changes can be identified in locomotion experiments on variable ground height. Grimmer and colleagues showed that leg stiffness and angle of attack are two control parameters to cope with ground level changes ([Grimmer et al., 2008](#)). A suitable control strategy to cope with ground level changes is to adapt the leg stiffness to an altered angle of attack. This adaptation is within the J-shaped area in the leg stiffness-attack angle space predicted for stable running in the SLIP model ([Seyfarth et al., 2002](#)).

Gravity as an external force influences locomotion control on level and inclined terrains. However, gravitational effects are more critical on sloped grounds or stairs. For example, the largest percentage of falls occurs during

stair walking in public places. Here, 80% of these falls on stairs relate to stair descent (Shumway-Cook and Woollacott, 2007). The stance phase in stair ascending is divided into three subphases: (i) weight acceptance, (ii) pull-up, and (iii) forward continuance. The swing phase is divided into (i) foot clearance and (ii) foot placement (Shumway-Cook and Woollacott, 2007), which is similar to level ground gait. The main contributors in foot clearance are tibialis anterior (for foot dorsiflexion) and hamstrings (for knee flexion) while rectus femoris contributes to the second half of the swing phase (similar to level walking Prilutsky and Gregor, 2001). The swing leg—guided by the movement of the pelvis—is lifted and moved forward by hip flexion followed by hip extension and ankle dorsiflexion in preparation of foot placement on the higher step (McFadyen and Winter, 1988). Reducing sensory information (e.g., visual feedback in blind-folded stair gaits) influences swing leg control strategy in walking on stairs much more than in level walking. For example, with limited visual feedback, anticipatory gastrocnemius activation is reduced and the leg is more compliant during landing in stair descending (Craik, 1982). With reduced visual sensory information (e.g., blurred-vision), foot clearance and foot placement become critical control strategies for stair descending. In such conditions, the foot is placed further backwards on the step to increase the safety margin (Simoneau et al., 1991).

2.2.2.4 Recovery from Perturbations

Strategies for perturbation recovery may be divided to three categories: (i) small perturbations that can be recovered by intrinsic muscle behavior (by damping property of the system dynamic) without requiring more activation, (ii) moderate perturbations requiring stance leg muscle activation for achieving posture balance, (iii) large perturbations which need stepping for compensating perturbation and returning to a stable solution (fixed point or limit cycle). In the latter group of perturbations, swing leg adjustment plays the most significant role. Velocity based leg adjustment (see Sect. 2.2.3) are the most common methods used to find the point (region) to place the foot for perturbation recovery of models and machines (robots). Another group is benefiting from system (passive) dynamics such as pendulum-like movement (McGeer, 1990a). After perturbation occurrence, pendulum-like passive dynamics yields shorter steps (than normal steps) which results in lower impacts, but still tolerable in for-aft direction (Kuo and Donelan, 2010).

Eng et al. have investigated the movement strategies and neuromuscular responses to recover from a tripping perturbation in humans (Eng et al., 1994). They have studied perturbations in early and late swing phase of walking. For early swing perturbations, the most common control strategy was an elevating the swing limb while the swing limb lowers in response to the late swing

perturbation. In the elevating strategy both swing limb flexion and stance limb extension are involved. Two goals are achieved by this control strategy. Firstly, it removes the limb from the obstacle prior to accelerating it over the obstacle. Secondly, the extensor response of the stance limb generates an early heel-off which increases the CoM height. This provides extra time to extend the swing limb in preparation for the landing. In contrast, swing leg flexion may be dangerous in late swing perturbations, because the swing limb is approaching the ground and the body mass is in front of the stance foot. Instead, the swing leg lowers rapidly (with a flat foot or forefoot landing) which shortens the step length. Hence, the similar recovery strategy by different patterns of muscle activation is generated in early and late perturbations (Eng et al., 1994).

2.2.3 SWING LEG MODELING AND CONTROL

Template models such as the inverted pendulum model (Cavagna et al., 1963; Cavagna and Margaria, 1966) and the spring–mass model (SLIP, spring-loaded inverted pendulum) (Blickhan, 1989; Full and Koditschek, 1999) can help understand principles inherent in human locomotion and to demonstrate them in robotic counterparts. These models concentrate on the description of ground reaction forces (GRF) and center of mass (CoM) trajectories and neglect the effects of swing leg dynamics. In the swing phase of walking, beside ground clearance, the main function of the swing leg is providing an appropriate foot placement, i.e., achieving a suitable leg configuration, a desired angle of attack, and leg retraction in preparation of the next contact phase. Although the swing leg mass also affects whole body motion, in most studies this effect is ignored. In these models, the focus is on COM dynamics and the representation of swing leg movement is reduced to describe an appropriate angle of attack (Kuo, 2007b; Knuesel et al., 2005). In the following, we present an overview of different swing leg adjustments based on such simplified models and we introduce a new model for leg placement based on leg dynamics (Mohammadi et al., 2014).

2.2.3.1 Massless Swing Leg

Using a fixed angle of attack with respect to the ground can stabilize running (Seyfarth et al., 2002) and walking (Geyer et al., 2006). However, the region of possible leg adjustments (regarding leg stiffness, leg angle) for the stable gait is quite limited. The next levels of swing leg adjustment approaches are (1) swing leg retraction with a given leg rotation speed (Herr et al., 2002; Seyfarth et al., 2003) (see Sect. 2.2.2) and (2) adapting the leg angle during leg swinging using state feedback (Pratt and Tedrake, 2006). Leg adjustment strategies may rely on sensory information about the CoM velocity (Raibert, 1986) in which the foot landing position is adjusted based on

the horizontal velocity (e.g., [Pratt et al., 2006](#); [Poulakakis and Grizzle, 2009](#); [Sato and Beuhler, 2004](#)). Peuker et al. concluded that leg placement with respect to both the CoM velocity and the gravity vectors yielded the most robust and stable hopping and running motions with the SLIP model ([Peuker et al., 2012](#)). As a modification of Peuker's approach, [Sharbafi et al. \(2013\)](#) developed a novel VBLA (velocity based leg adjustment) controller. In this controller, the velocity vector (with horizontal and vertical components, \dot{x} and \dot{y}) is used to adjust the angle of attack:

$$\alpha = \arctg\left(\frac{\dot{y} - c\sqrt{l}g}{\dot{x}}\right) = \arctg\left(\frac{\dot{y} - kg}{\dot{x}}\right), \quad (2.2.2)$$

in which α , l , and g are the angle of attack (with respect to ground), leg length (hip-to-foot point), and gravitational acceleration, respectively. Here, c is a dimensionless tuning parameter (gain) which can be lumped into the parameter $k = c\sqrt{l}g$. A comparison of the three methods (VBLA, Raibert and Peuker approaches) showed that VBLA better mimics human-like leg adjustment in perturbed hopping in place, achieves the largest range of running velocities by a fixed set of control parameters, and predicts robust walking in a bipedal SLIP model with extended rigid trunk ([Sharbafi and Seyfarth, 2016](#)). This method was successfully implemented on a simulation model of a bioinspired robot (called BioBiped) to generate stable forward hopping with adjustable speeds ([Sharbafi et al., 2014](#)). The idea of zeroing horizontal speed (like in recovery from perturbed hopping) with an appropriate swing leg adjustment in walking was presented within the capture point concept ([Pratt and Tedrake, 2006](#)). This approach determines a position for foot placement to stop forward motion. For a bipedal system, *capture state* is defined as the state with zero kinetic energy level. By placing the foot (CoP) on a *capture point* P , the controlled motion dynamics moves the states to reach the capture state. The set of all capture points is called capture region. These concepts can be extended to n -step capture point and n -step capture region using leg swinging and recursive definition ([Pratt and Tedrake, 2006](#)). If n approaches ∞ , the n -step walking can be achieved because the capture region converges to the area on the ground that the foot can be placed at without falling. For implementation on robots, usually simple models like inverted pendulum ([Pratt and Tedrake, 2006](#)) or linear inverted pendulum ([Pratt et al., 2006](#)) are utilized to find the capture point analytically.

2.2.3.2 Mass in the Swing Leg

Mochon and McMahon presented a model comprising a stiff stance leg and a segmented swing leg ([Mochon and McMahon, 1980](#)). Compared with the inverted pendulum model, this model provides a better match of human walking

dynamics. Introducing the spring loaded inverted pendulum (SLIP) model, the gait dynamics (GRF and COM movement) of human locomotion can be better represented (Geyer et al., 2006) compared to the inverted pendulum model. However, swing leg movement is still a missing part in SLIP based models. In Sharbafi et al. (2017) a segmented swing leg is added to the SLIP model to represent the swing phase of human-like walking. Such template models help to better understand key features of human walking (e.g. muscle activation patterns and segment motions) which could previously be observed in more complex gait models (Geyer and Herr, 2010).

Judging from human leg muscle activities in the swing leg movement, biarticular hip muscles rectus femoris (RF) and hamstrings (HA) seem to be main contributors for swing leg control in the swing phase of walking (Nilsson et al., 1985). By modeling these two muscles with biarticular springs, better mechanical understanding of their activities in producing stable gait is obtained. In addition, such a passive mechanism may also replicate strong correlation observed between RF and HA in human swing leg movement (Prilutsky et al., 1998), as a consequence of body mechanics. The role of elastic biarticular thigh muscles (represented as springs) on swing leg dynamics can be further investigated, and the appropriate spring parameters and morphology can mimic human swing leg motion in walking. The muscle lever arm ratio, muscle stiffness, and muscle rest lengths influence the COM motion and swing leg behavior. With passive elastic biarticular muscles, walking motion characteristics like swing leg retraction and symmetric stance leg behavior around mid-stance are predicted (Mohammadi et al., 2014). Such a simple bio-inspired control approach can be implemented in robots (Sharbafi et al., 2016). During the swing phase, biarticular muscles can support swing leg rotational movement control while monoarticular muscles (e.g., knee or ankle joints) can provide (axial) leg shortening and lengthening (e.g., leg shortening is required for ground clearance). With such a muscle-specific task allocation, the target of control could be simply setting spring rest lengths to a specific value for each gait condition.

This simple control strategy is able to produce human-like forces and kinematic behavior in walking. It was successfully approved in a simulation model of BioBiped robot for describing forward hopping (Sharbafi et al., 2016). Changing the motion speed can be achieved by adjusting the rest angle of biarticular springs. This provides a simple and efficient swing leg control approach without needing sensory information of the leg configuration. In order to achieve high efficiency during different phases of the gait cycle (e.g., swing phase), non-backdrivable actuators are advantageous. They enable setting the springs' rest lengths to desired values and switching off the motors and to operate with no (or little) resistance when no actuation is needed.

For stable locomotion, swing-leg adjustment needs to complement the other locomotor subfunctions (stance and balance). In future, a better understanding of the interplay of these subfunctions needs to be developed. These insights will help to further improve the design and modular control of locomotor systems both in simulation models and in hardware.

REFERENCES

- Alexander, R.McN., 1984. The gaits of bipedal and quadrupedal animals. *Int. J. Robot. Res.* 3 (2), 49–59.
- Bauby, C.E., Kuo, A.D., 2000. Active control of lateral balance in human walking. *J. Biomech.* 33 (11), 1433–1440.
- Birn-Jeffery, A., Daley, M.A., 2012. Birds achieve high robustness in uneven terrain through active control of landing conditions. *J. Exp. Biol.* 215, 2117–2127.
- Begg, Rezaul, Best, Russell, Dell’Oro, Lisa, Taylor, Simon, 2007. Minimum foot clearance during walking: strategies for the minimisation of trip-related falls. *Gait Posture* 25 (2), 191–198.
- Blickhan, R., 1989. The spring–mass model for running and hopping. *J. Biomech.* 22 (11), 1217–1227.
- Blum, Y., Lipfert, S.W., Rummel, J., Seyfarth, A., 2010. Swing leg control in human running. *Bioinspir. Biomim.* 5, 026006.
- Cavagna, G., Saibene, F., Margaria, R., 1963. External work in walking. *J. Appl. Physiol.* 18 (1), 1–9.
- Cavagna, G., Margaria, R., 1966. Mechanics of walking. *J. Appl. Physiol.* 21 (1), 271–278.
- Collins, S., Ruina, A., Tedrake, R., Wisse, M., 2005. Efficient bipedal robots based on passive-dynamic walkers. *Science* 307 (5712), 1082–1085.
- Craik, R.L., 1982. Clinical correlates of neural plasticity. *Phys. Ther.* 62 (10), 1452–1462.
- Daley, M.A., Biewener, A.A., 2006. Running over rough terrain reveals limb control for intrinsic stability. *Proc. Natl. Acad. Sci. USA* 103, 15681–15686.
- Daley, M.A., Felix, G., Biewener, A.A., 2007. Running stability is enhanced by a proximo-distal gradient in joint neuromechanical control. *J. Exp. Biol.* 210 (3), 383–394.
- Daley, M.A., Biewener, A.A., 2011. Leg muscles that mediate stability: mechanics and control of two distal extensor muscles during obstacle negotiation in the Guinea fowl. *Philos. Trans. R. Soc. B* 366, 1580–1591.
- De Wit, B., De Clercq, D., Aerts, P., 2000. Biomechanical analysis of the stance phase during bare-foot and shod running. *J. Biomech.* 33, 269–278.
- Doke, J., Donelan, J.M., Kuo, A.D., 2005. Mechanics and energetics of swinging the human leg. *J. Exp. Biol.* 208 (3), 439–445.
- Donelan, J.M., Kram, R., Kuo, A.D., 2002. Mechanical work for step-to-step transitions is a major determinant of the metabolic cost of human walking. *J. Exp. Biol.* 205, 3717–3727.
- Dunn, E.R., Howe, R.D., 1996. Foot placement and velocity control in smooth bipedal walking. In: *Proceedings, 1996 IEEE International Conference on Robotics and Automation*, vol. 1. IEEE, pp. 578–583.
- Eng, Janice J., Winter, David A., Patla, Aftab E., 1994. Strategies for recovery from a trip in early and late swing during human walking. *Exp. Brain Res.* 102 (2), 339–349.
- Full, R.J., Koditschek, D., 1999. Templates and anchors: neuromechanical hypotheses of legged locomotion on land. *J. Exp. Biol.* 22, 3325–3332.
- Geyer, H., Seyfarth, A., Blickhan, R., 2006. Compliant leg behaviour explains basic dynamics of walking and running. *Proc. R. Soc. B* 273 (1603), 2861–2867.

- Geyer, H., Herr, H., 2010. A muscle-reflex model that encodes principles of legged mechanics produces human walking dynamics and muscle activities. *IEEE Trans. Neural Syst. Rehabil. Eng.* 18 (3), 263–273.
- Gray, J., 1968. *Animal Locomotion*. Weidenfeld & Nicolson.
- Grimmer, S., Ernst, M., Guenther, M., Blickhan, R., 2008. Running on uneven ground: leg adjustment to vertical steps and self-stability. *J. Exp. Biol.* 211 (18), 2989–3000.
- Herr, H.M., McMahon, T.A., 2001. A galloping horse model. *Int. J. Robot. Res.* 20, 26–37.
- Herr, H.M., Huang, G., McMahon, T.A., 2002. A model of scale effects in mammalian quadrupedal running. *J. Exp. Biol.* 205, 959–967.
- Hreljac, A., 1993. Preferred and energetically optimal gait transition speeds in human locomotion. *Med. Sci. Sports Exerc.* 25, 1158–1162.
- Jian, Y., Winter, D.A., Ishac, M.G., Gilchrist, L., 1993. Trajectory of the body COG and COP during initiation and termination of gait. *Gait Posture* 1, 9–22.
- Knuesel, H., Geyer, H., Seyfarth, A., 2005. Influence of swing leg movement on running stability. *Hum. Mov. Sci.* 24, 532–543.
- Kuo, A.D., 1999. Stabilization of lateral motion in passive dynamic walking. *Int. J. Robot. Res.* 18 (9), 917–930.
- Kuo, A.D., 2002. Energetics of actively powered locomotion using the simplest walking model. *J. Biomech. Eng.* 124 (1), 113–120.
- Kuo, A.D., 2007a. The six determinants of gait and the inverted pendulum analogy: a dynamic walking perspective. *Hum. Mov. Sci.* 26 (4), 617–656.
- Kuo, A.D., 2007b. Choosing your steps carefully. *IEEE Robot. Autom. Mag.* 14 (2), 18–29.
- Kuo, Arthur D., Donelan, J. Maxwell, 2010. Dynamic principles of gait and their clinical implications. *Phys. Ther.* 90 (2), 157–174.
- Lieberman, Daniel E., Venkadesan, Madhusudhan, Werbel, William A., Daoud, Adam I., D’Andrea, Susan, Davis, Irene S., Mang’Eni, Robert Ojiambo, Pitsiladis, Yannis, 2010. Foot strike patterns and collision forces in habitually barefoot versus shod runners. *Nature* 463 (7280), 531–535.
- Long, L., Srinivasan, M., 2013. Walking, running and resting under time, distance, and speed constraints: optimality of walk-run-rest mixtures. *J. R. Soc. Interface* 10.
- MacKinnon, C.D., Winter, D.A., 1993. Control of whole body balance in the frontal plane during human walking. *J. Biomech.* 26, 633–644.
- Maus, H.M., Seyfarth, A., 2014. Walking in circles: a modelling approach. *J. R. Soc. Interface* 11 (99), 20140594.
- McFadyen, B.J., Winter, D.A., 1988. An integrated biomechanical analysis of normal stair ascent and descent. *J. Biomech.* 21 (9), 733–744.
- McGeer, T., 1990a. Passive dynamic walking. *Int. J. Robot. Res.* 9 (3), 62–82.
- McGeer, T., 1990b. Passive walking with knees. In: *Proc. IEEE Int. Robotics Automation Conf.* Los Alamitos, CA, pp. 1640–1645.
- McVea, D.A., Pearson, K.G., 2006. Long-lasting memories of obstacles guide leg movements in the walking cat. *J. Neurosci.* 26 (4), 1175–1178.
- Mills, Peter M., Barrett, Rod S., Morrison, Steven, 2008. Toe clearance variability during walking in young and elderly men. *Gait Posture* 28 (1), 101–107.
- Miura, H., Shimoyama, I., 1984. Dynamic walking of a biped. *Int. J. Robot. Res.* 3 (2), 60–74.
- Mochon, S., McMahon, T.A., 1980. Ballistic walking. *J. Biomech.* 13, 49–75.
- Mohammadi, A., Sharbafi, M.A., Seyfarth, A., 2014. SLIP with swing leg augmentation as a model for running. In: *2014 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, pp. 2543–2549.
- Moritz, C.T., Farley, C.T., 2004. Passive dynamics change leg mechanics for an unexpected surface during human hopping. *J. Appl. Physiol.* 97, 1313–1322.
- Muybridge, E., 1955. *The Human Figure in Motion*.

- Neptune, R.R., Sasaki, K., 2005. Ankle plantar flexor force production is an important determinant of the preferred walk-to-run transition speed. *J. Exp. Biol.* 208 (5), 799–808.
- Neptune, R.R., Sasaki, K., Kautz, S.A., 2008. The effect of walking speed on muscle function and mechanical energetics. *Gait Posture* 28 (1), 135–143.
- Nilsson, J., Thorstensson, A., Halbertsma, J., 1985. Changes in leg movements and muscle activity with speed of locomotion and mode of progression in humans. *Acta Physiol. Scand.* 123 (4), 457–475.
- Peuker, F., Maufroy, C., Seyfarth, A., 2012. Leg adjustment strategies for stable running in three dimensions. *Bioinspir. Biomim.* 7, 036002.
- Poggensee, K.L., Sharbafi, M.A., Seyfarth, A., 2014. Characterizing swing leg retraction in human locomotion. In: *International Conference on Climbing and Walking Robots (CLAWAR 2014)*.
- Poulakakis, I., Grizzle, J.W., 2009. The spring loaded inverted pendulum as the hybrid zero dynamics of an asymmetric hopper. *IEEE Trans. Autom. Control* 54, 1779–1793.
- Pratt, J.E., Tedrake, R., 2006. Velocity-based stability margins for fast bipedal walking. In: *Fast Motions in Biomechanics and Robotics*. Springer, Berlin, Heidelberg, pp. 299–324.
- Pratt, J., Carff, J., Drakunov, S., Goswami, A., 2006. Capture point: a step toward humanoid push recovery. In: *6th IEEE–RAS International Conference on Humanoid Robots*, pp. 200–207.
- Prilutsky, B.I., Gregor, R.J., Ryan, M.M., 1998. Coordination of two-joint rectus femoris and hamstrings during the swing phase of human walking and running. *Exp. Brain Res.* 120 (4), 479–486.
- Prilutsky, B.I., Gregor, R.J., 2001. Swing-and support-related muscle actions differentially trigger human walk–run and run–walk transitions. *J. Exp. Biol.* 204 (13), 2277–2287.
- Raibert, M., 1986. *Legged Robots That Balance*. MIT Press, Cambridge, MA.
- Sakagami, Y., Watanabe, R., Aoyama, C., Matsunaga, S., Higaki, N., Fujimura, K., 2002. The intelligent ASIMO: system overview and integration. In: *Proc. IEEE/RSJ Int. Conf. Intelligent Robots Systems*, pp. 2478–2483.
- Sato, A., Beuhler, M., 2004. A planar hopping robot with one actuator: design, simulation, and experimental results IROS. In: *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*.
- Segers, V., Aerts, P., Lenoir, M., De Clercq, D., 2006. Spatiotemporal characteristics of the walk-to-run and run-to-walk transition when gradually changing speed. *Gait Posture* 24 (2), 247–254.
- Seyfarth, A., Günther, M., Blickhan, R., 2001. Stable operation of an elastic three-segment leg. *Biol. Cybern.* 84 (5), 365–382.
- Seyfarth, A., Geyer, H., Günther, M., Blickhan, R., 2002. A movement criterion for running. *J. Biomech.* 35 (5), 649–655.
- Seyfarth, A., Geyer, H., Herr, H., 2003. Swing-leg retraction: a simple control model for stable running. *J. Exp. Biol.* 206, 2547–2555.
- Sharbafi, M.A., Maufroy, C., Ahmadbadi, M.N., Yazdanpanah, M.J., Seyfarth, A., 2013. Robust hopping based on virtual pendulum posture control. *Bioinspir. Biomim.* 8 (3), 036002.
- Sharbafi, M.A., Radkhah, K., von Stryk, O., Seyfarth, A., 2014. Hopping control for the musculoskeletal bipedal robot: BioBiped. In: *2014 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 4868–4875.
- Sharbafi, M.A., Rode, C., Kurowski, S., Scholz, D., Möckel, R., Radkhah, K., Zhao, G., Mohammad, A., von Stryk, O., Seyfarth, A., 2016. A new biarticular actuator design facilitates control of leg function in BioBiped3. *Bioinspir. Biomim.* 11 (4), 046003.
- Sharbafi, M.A., Mohammadinejad, A., Rode, C., Seyfarth, A., 2017. Reconstruction of human swing leg motion with passive biarticular muscle models. *Human Movement Science* 52, 96–107.
- Sharbafi, M.A., Seyfarth, A., 2016. VBLA, a swing leg control approach for humans and robots. In: *IEEE-RAS International Conference on Humanoid Robots (Humanoids 2016)*.
- Sharbafi, M.A., Seyfarth, A., 2017. How locomotion sub-functions can control walking at different speeds? *J. Biomech.* 53, 163–170.

- Shumway-Cook, A., Woollacott, M.H., 2007. *Motor Control: Translating Research Into Clinical Practice*. Lippincott Williams & Wilkins.
- Siegler, R., Deloache, J., Eisenberg, N., 2006. *How Children Develop*. Worth Publishers, New York. ISBN 978-0-7167-9527-8, p. 188.
- Simoneau, G.G., Cavanagh, P.R., Ulbrecht, J.S., Leibowitz, H.W., Tyrrell, R.A., 1991. The influence of visual factors on fall-related kinematic variables during stair descent by older women. *J. Gerontol.* 46 (6), M188–M195.
- Thorstensson, A., Robertson, H., 1987. Adaptations to changing speed in human locomotion: speed of transition between walking and running. *Acta Physiol. Scand.* 131, 211–214. <http://dx.doi.org/10.1111/j.1748-1716.1987.tb08228.x>.
- Townsend, M.A., 1985. Biped gait stabilization via foot placement. *J. Biomech.* 18, 21–38.
- Vejdani, H.R., Blum, Y., Daley, M.A., Hurst, J.W., 2013. Bio-inspired swing leg control for spring-mass robots running on ground with unexpected height disturbance. *Bioinspir. Biomim.* 8 (4), 046006.
- Voloshina, A.S., Kuo, A.D., Daley, M.A., Ferris, D.P., 2013. Biomechanics and energetics of walking on uneven terrain. *J. Exp. Biol.* 216, 3963–3970.
- Vukobratovic, M., Borovac, B., 2004. Zero-moment point-thirty-five years of its life. *Int. J. Humanoid Robot.* 1 (1), 157–173.
- Warren, W.H., Kay, B.A., Yilmaz, E.H., 1996. Visual control of posture during walking: functional specificity. *J. Exp. Psychol. Hum. Percept. Perform.* 22, 818–838.
- Winter, David A., 1992. Foot trajectory in human gait: a precise and multifactorial motor control task. *Phys. Ther.* 72 (1), 45–53.
- Winter, D., 1995. Human balance and posture control during standing and walking. *Gait Posture* 3, 193–214.
- Wisse, M., Schwab, A.L., van der Linde, R.Q., van der Helm, F.C.T., 2005. How to keep from falling forward; elementary swing leg action for passive dynamic walkers. *IEEE Trans. Robot.* 21 (3), 393–401.
- Wu, A.R., Kuo, A.D., 2015. Energetic tradeoffs of foot-to-ground clearance during swing phase of walking. In: *Dynamic Walking*. Columbus Ohio, USA.

Chapter 2.3

Balancing

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Balance is the control of the body's orientation relative to vertical. To describe the mechanics of balance, we start with some definitions (Vukobratović and Borovac, 2004; Vukobratović et al., 2006; Herr and Popovic, 2008; Maus et al., 2010; Xiang et al., 2010; Goswami, 1999). Balance depends on the *support base*, the smallest convex set on the support surface containing all contact points between the feet and support surface. As an approximation, consider a finite number of contact points \mathbf{q}_i with forces \mathbf{f}_i acting on the feet (Fig. 2.3.1A). Let $f_{ni}\hat{\mathbf{z}}$ be the component of \mathbf{q}_i normal to the support surface, where $\hat{\mathbf{z}}$ is the vertical basis vector. Define the *center of pressure* (COP) as

$$\text{COP} = \frac{\sum f_{ni}\mathbf{q}_i}{\sum f_{ni}} \quad (2.3.1)$$

and the *ground reaction force* (GRF) as $\sum \mathbf{f}_i$. Then the net effect of all contact forces \mathbf{f}_i acting at \mathbf{q}_i equals the GRF acting at the COP and a moment \mathbf{M} about the COP. The horizontal components of \mathbf{M} are zero (Goswami, 1999), so the COP is often referred to in the robotics literature as the *zero-moment point* (ZMP). Some authors consider the COP and ZMP to be equivalent (e.g., Goswami, 1999). However, others consider the ZMP to be defined only when the COP is in the interior of the support based (e.g., Vukobratović and Borovac, 2004; Vukobratović et al., 2006) (Fig. 2.3.1A), since the foot may rotate about the COP when the COP lies on the boundary of the support base (Fig. 2.3.1B). To describe such foot rotation, one can define an extension of the ZMP, known as the *foot rotation index* (FRI) (Goswami, 1999) or *fictitious zero-moment point* (FZMP) (Vukobratović and Borovac, 2004), as the point on the support surface where the GRF would need to act to prevent foot rotation. When the FRI/ZMP lies in the interior of the support base, then it coincides with the ZMP = COP and there is no foot rotation (Fig. 2.3.1A).

With these definitions, we can understand the mechanics of balance in terms of how the GRF acting at the COP changes the body's angular momentum $\mathbf{L}(t)$ about its center of mass (COM). Herr and Popovic (2008) describe this relationship using the *centroidal moment pivot* (CMP), as illustrated in Fig. 2.3.2 for sagittal-plane motion. The CMP is the intersection of the support surface and the line parallel to the GRF that passes through the COM. For sagittal-plane motion, change in angular momentum $dL_x(t)/dt$ equals the normal component of

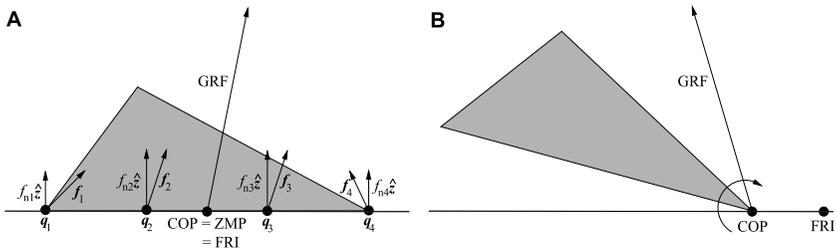


FIGURE 2.3.1 (A) The center of pressure (COP) lies in the interior of the base of support and, thus, coincides with the zero moment point (ZMP) and foot rotation index (FRI). (B) The COP lies on the boundary of the base of support and $FRI \neq COP$ indicates that the foot will rotate about the COP.

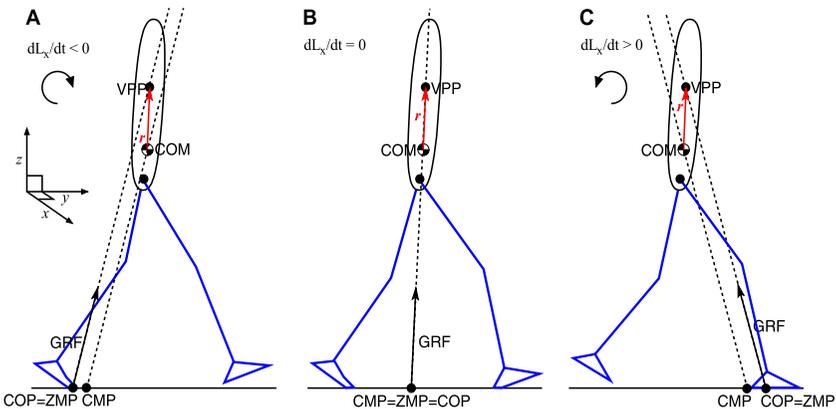


FIGURE 2.3.2 Whole-body mechanics of balance in the sagittal plane.

the GRF times the position of the COP relative to the CMP. Angular momentum changes in the forward (clockwise) or backward (counterclockwise direction) depending on whether the COP is behind the CMP (Fig. 2.3.2A) or ahead of the CMP (Fig. 2.3.2C), respectively.

For periodic locomotion, both $L_x(t)$ and $dL_x(t)/dt$ are periodic with mean 0, allowing angular momentum $L_x(t)$ to be uniquely computed by integrating $dL_x(t)/dt$ (Herr and Popovic, 2008; Elftman, 1939). For human walking, $L_x(t)$ is backwards during mid-stance and increases in the forward direction near the end of single-support when the COP is behind the CMP (Fig. 2.3.2A). $L_x(t)$ obtains its greatest forwards value near heel strike when $dL_x(t)/dt = 0$ (Fig. 2.3.1B). $L_x(t)$ changes in the backward direction when the COP is ahead of the CMP after heel strike (Fig. 2.3.2C), and is again backwards by mid-stance of the opposite leg. This pattern leads to a cancellation of positive and negative values of $L_x(t)$ so that it has mean 0 over the gait cycle. Such temporal cancella-

tion must occur for any periodic gait. In addition, for human walking, there is cancellation of the angular momenta of different body segments at each point in time so that $L_x(t)$ is small throughout the gait cycle (Herr and Popovic, 2008). In other words, the COP and CMT remain close throughout gait cycle, leading to “zero-moment control”.

Another method to describe the relationship between the GRF and the COM involves the *virtual pivot point* (VPP) (Maus et al., 2010). In the strictest definition, as illustrated in Fig. 2.3.2, the VPP is a point on the body with a fixed position relative to the COM (i.e., the vector \mathbf{r} is constant) such that at each point in time, the line through the COP in the direction of the GRF passes through this point. For human walking, there is a VPP above the COM that approximately meets this definition (Maus et al., 2010) (Fig. 2.3.2). Maus et al. (2010) propose a conceptual model in which the VPP above the COM acts like the pivot point of a virtual pendulum, leading to stable balance during locomotion.

Comparing the approaches of Herr and Popovic (2008) and Maus et al. (2010), both examine the relationship between the GRF and the COM. Herr and Popovic (2008) emphasize that the line of action of the GRF passes close to the COM throughout the gait cycle, that is, there is approximate zero-moment control. If zero-moment control were perfect, then the VPP would be the COM. Instead, the small deviations from zero-moment control are such that the VPP is above the COM, leading Maus et al. (2010) to propose virtual pendulum control to explain stable balance during gait.

2.3.1 THE NEURAL CONTROL OF BALANCE: STANDING VS. WALKING

Balance has been extensively studied for standing (Horak and Macpherson, 2010), where the base of support is fixed. The nervous system uses information from various sensory systems (somatosensation, vision, and the vestibular system) to detect deviations away from the desired (nearly) vertical of various body segments, such as the trunk, thighs, and shanks. The nervous system corrects these deviations by modulating the stimulation of muscles acting at various joints, such as the ankles, knees, and hips. Balance during locomotion shares many of the same basic features, although there are important differences due to the changing base of support and rhythmicity inherent in locomotion. Here we consider these similarities and differences, focusing on the example of the control of the trunk relative to vertical.

Fig. 2.3.3 is a schematic representation of the neural control of balance in the general framework of control theory. Here the plant, the entity being controlled, is an input–output process that describes how stimulation of muscles by the nervous system leads to movement. The plant is defined by the biomechanical properties of the body and its mechanical interaction with the environment.

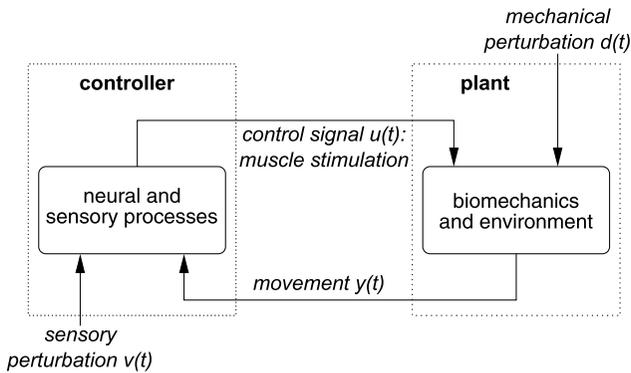


FIGURE 2.3.3 Schematic diagram of the neural control of movement, including standing balance and balance during locomotion.

The controller describes how the nervous system uses sensory information to modulate the stimulation of muscles. Balance is the result of the closed-loop interaction between the plant and the controller.

There are two basic questions about the neural control of balance. First, what is the desired attractor corresponding to stable balance? Second, how does neural feedback stabilize this attractor? For standing, the attractor is a stable fixed point and, thus, is easy to describe: the body, on average, leans slightly forward and there is, on average, a low level of stimulation of appropriate muscles (such as calf muscles) to counteract the small torques due to gravity resulting from the slight forward lean. For walking the attractor is a limit cycle. One must describe, as a function of the phase of gait cycle, the periodic stimulation of each muscle and the periodic movement of each mechanical degree of freedom of the body. For example, Fig. 2.3.4A shows the periodic stimulation, as measured by surface electromyography (EMG), of the erector spinae muscle of the lower trunk, which acts to rotate the trunk backwards and is one of many muscles involved in maintaining trunk orientation (Logan et al., 2017). Similarly, Fig. 2.3.4E shows the periodic motion of the trunk in the sagittal plane. The closed-loop nature of neural control implies that there is a consistency between EMG and kinematic waveforms. The periodic EMG waveforms of all muscles and the properties of the plant predict the periodic waveforms of each mechanical degree of freedom. Conversely, the periodic waveforms of all mechanical degrees of freedom and the properties of the neural controller predict the periodic waveform of each muscle.

One early approach to understanding the relationship among mean EMG and kinematic waveforms was the concept of balancing and unbalancing moments of Winter (1995). The vertical and especially horizontal acceleration of the hip

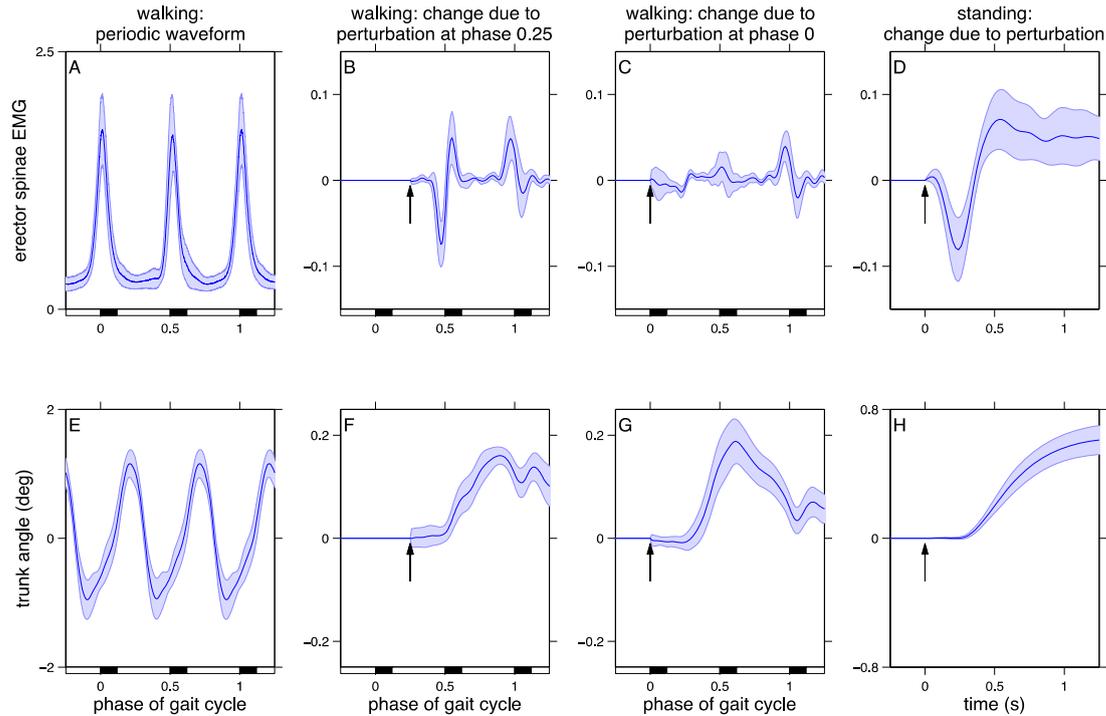


FIGURE 2.3.4 Walking and standing perturbed by movement of the visual scene. Responses to continuous movement of the visual scene were used to infer the effect of a small step in visual-scene position at the times indicated by arrows. Black bars along horizontal axes indicate double-support phases of the gait cycle. Based on data from [Kiemel et al. \(2011\)](#) for standing and [Logan et al. \(2017\)](#) for walking.

during walking produce an “unbalancing movement” of the trunk about the hip. Muscles must provide a counteracting “balancing” moment to keep the amplitude of trunk-angle oscillation within the observed range of about a degree or two (Fig. 2.3.4E). For example, after heel strike there is a backwards acceleration of the hip, which if not counteracted would produce a large forward rotation of the hip. The stimulation of erector spinae and other trunk extensor muscles near heel strike (Fig. 2.3.4A) produce a counteracting backwards moment on the trunk. Note that stimulation of the erector spinae muscle starts before heel strike, suggesting that the nervous system anticipates the unbalancing moment rather than merely reacting to its effects.

Another approach to understanding the relationship among mean EMG and kinematic waveforms is optimal control modeling (Anderson and Pandy, 2001; Miller, 2014). Given a model of the plant and a cost function, periodic EMG and kinematic waveforms are found that minimize the cost function for walking at a specified speed. The cost function typically penalizes metabolic cost and/or the amount of muscle stimulation. The amount of trunk accelerations predicted by these models roughly approximates observed behavior, although their cost functions do not penalize acceleration. This calls into question the common belief that balance during locomotion is designed to reduce head accelerations in order to improve visual and vestibular sensing (Winter, 1995).

Sensory feedback control

The trunk during locomotion, as during standing, acts like an inverted pendulum (Winter, 1995) and, thus, is unstable without sensory feedback. Given the wealth of knowledge about how sensory information is used to stabilize standing balance, it is advantageous to consider whether similar principles apply to locomotion, which is much less studied. In particular, system identification methods based on sensory and mechanical perturbations have been used to identify the key features of the plant and the neural controller (Fig. 2.3.2) (van der Kooij et al., 2005; Kiemel et al., 2011) for standing balance. By comparing responses to such perturbations during standing and locomotion, we can gain insight into the similarities and differences between the two types of balance. For example, if the visual scene moves forward, a standing subject interprets this environmental motion as self-motion in the backwards direction. As a result, the nervous system changes levels of muscle stimulation, such as reducing the stimulation of the erector spinae muscles (Fig. 2.3.4D), in order to rotate the trunk forward (Fig. 2.3.4H). The reduction in stimulation is followed by an increase, which acts to limit the forward trunk rotation and eventually, on a longer time scale than shown in Fig. 2.3.4H, bring the trunk back to its original orientation.

If the same visual-scene perturbation is applied to a walking subject early during the single-support phase of the gait cycle, the initial response is similar:

a decrease in erector spinae stimulation (Fig. 2.3.4B) followed by a forward trunk rotation (Fig. 2.3.4F). However, the response is highly dependent on the phase of the gait cycle at which the perturbation occurs. For example, if the perturbation occurs at the beginning of double support, then the initial erector spinal response is greatly reduced (Fig. 2.3.4C). Instead, the stimulations of other muscles respond to the perturbation (not shown), so that the trunk still rotates forward (Fig. 2.3.4G).

This phase dependence occurs because the change in a muscle's stimulation due to a small perturbation occurs during those phases of the gait cycle when the muscle is normally stimulated. For example, the effects of perturbations on erector spinae stimulation occur around the beginning of double support (Fig. 2.3.4B, C) when the periodic stimulation is highest (Fig. 2.3.4A). Thus, the set of muscles that the nervous system can use to provide the earliest response to a perturbation depends on the phase at which the perturbation occurs. If a perturbation occurs during early double support, the erector spinae is in that early-response set. If the perturbation occurs at the beginning of double support, it is not and other muscles are used instead.

In summary, what emerges from the comparison of standing balance and locomotor balance is that both use similar feedback control mechanisms based on sensory information to respond to perturbations, but that locomotor balance has the additional property of phase dependence. This view of feedback control of locomotor balance is generally consistent with how feedback balance control has been implemented in walking models (Aoi et al., 2010; Geyer and Herr, 2010; Song and Geyer, 2015; Günther and Ruder, 2003). In models with a single trunk segment, the position and angular velocity of the trunk is used modulate the stimulation of muscles that act at the hip, but only when the given leg is in stance, giving rise to phase dependency.

REFERENCES

- Anderson, F.C., Pandy, M.G., 2001. Dynamic optimization of human walking. *J. Biomech. Eng.* 123, 381–390. <http://dx.doi.org/10.1115/1.1392310>.
- Aoi, S., Ogihara, N., Funato, T., Sugimoto, Y., Tsuchiya, K., 2010. Evaluating functional roles of phase resetting in generation of adaptive human bipedal walking with a physiologically based model of the spinal pattern generator. *Biol. Cybern.* 102, 373–387. <http://dx.doi.org/10.1007/s00422-010-0373-y>.
- Eftman, H., 1939. The function of the arms in walking. *Hum. Biol.* 11, 529–535.
- Geyer, H., Herr, H., 2010. A muscle-reflex model that encodes principles of legged mechanics produces human walking dynamics and muscle activities. *IEEE Trans. Neural Syst. Rehabil. Eng.* 18, 263–273. <http://dx.doi.org/10.1109/TNSRE.2010.2047592>.
- Goswami, A., 1999. Postural stability of biped robots and the Foot-Rotation Indicator (FRI) Point. *Int. J. Robot. Res.* 18, 523–533. <http://dx.doi.org/10.1177/02783649922066376>.
- Günther, M., Ruder, H., 2003. Synthesis of two-dimensional human walking: a test of the λ -model. *Biol. Cybern.* 89, 89–106.

- Herr, H., Popovic, M., 2008. Angular momentum in human walking. *J. Exp. Biol.* 211, 467–481. <http://dx.doi.org/10.1242/jeb.008573>.
- Horak, F.B., Macpherson, J.M., 2010. Postural orientation and equilibrium. In: *Comprehensive Physiology*. John Wiley & Sons, Inc. [cited 2015 Nov. 29].
- Kiemel, T., Zhang, Y., Jeka, J.J., 2011. Identification of neural feedback for upright stance in humans: stabilization rather than sway minimization. *J. Neurosci.* 31, 15144–15153. <http://dx.doi.org/10.1523/JNEUROSCI.1013-11.2011>.
- Logan, D., Kiemel, T., Jeka, J.J., 2017. Using a system identification approach to investigate subtask control during human locomotion. *Front. Comput. Neurosci.* 10. <http://dx.doi.org/10.3389/fncom.2016.00146>.
- Maus, H.-M., Lipfert, S.W., Gross, M., Rummel, J., Seyfarth, A., 2010. Upright human gait did not provide a major mechanical challenge for our ancestors. *Nat. Commun.* 1, 70. <http://dx.doi.org/10.1038/ncomms1073>.
- Miller, R.H., 2014. A comparison of muscle energy models for simulating human walking in three dimensions. *J. Biomech.* 47, 1373–1381. <http://dx.doi.org/10.1016/j.jbiomech.2014.01.049>.
- Song, S., Geyer, H., 2015. A neural circuitry that emphasizes spinal feedback generates diverse behaviours of human locomotion. *J. Physiol.* 593, 3493–3511. <http://dx.doi.org/10.1113/JP270228>.
- van der Kooij, H., van Asseldonk, E., van der Helm, F.C.T., 2005. Comparison of different methods to identify and quantify balance control. *J. Neurosci. Methods* 145, 175–203. <http://dx.doi.org/10.1016/j.jneumeth.2005.01.003>.
- Vukobratović, M., Borovac, B., 2004. Zero-moment point—thirty five years of its life. *Int. J. Humanoid Robot.* 01, 157–173. <http://dx.doi.org/10.1142/S0219843604000083>.
- Vukobratović, M., Borovac, B., Potkonjak, V., 2006. ZMP: a review of some basic misunderstandings. *Int. J. Humanoid Robot.* 03, 153–175. <http://dx.doi.org/10.1142/S0219843606000710>.
- Winter, D., 1995. Human balance and posture control during standing and walking. *Gait Posture* 3, 193–214. [http://dx.doi.org/10.1016/0966-6362\(96\)82849-9](http://dx.doi.org/10.1016/0966-6362(96)82849-9).
- Xiang, Y., Arora, J.S., Abdel-Malek, K., 2010. Physics-based modeling and simulation of human walking: a review of optimization-based and other approaches. *Struct. Multidiscip. Optim.* 42, 1–23. <http://dx.doi.org/10.1007/s00158-010-0496-8>.